SEMI-EMPIRICAL APPROACH TO THE DESCRIPTION OF GROUND-STATE ROTATIONAL BANDS OF DEFORMED EVEN-EVEN NUCLEI

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ABSTRACT

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ABSTRACT

A semi-empirical model is proposed which permits an excellent fit of level energies of ground-state bands in deformed even-even nuclei. In this model, the rotational energy is expressed as a polynomial in ω^2, ω^3, ω^4 (ω is the nuclear angular velocity). Each nucleus is thus described by three adjustable parameters, $\mathcal{I}_o$, B, and C, which are determined by a least-squares fit of all the known levels. The calculated energy levels and parameters are tabulated for 88 even-even nuclei. The range of validity of the model is discussed and compared to that of the VMI model. The role of the parameter B is shown to be that of modifying the nuclear softness, σ. In contrast to the VMI model, it is shown that negative values of σ may exist for $R_4 > 2.3$, and may be obtained without requiring a negative ground-state moment of inertia.

1. INTRODUCTION

In recent years the problem of the prediction of the nuclear rotational energy levels has attracted considerable interest. One of the main reasons is that the development of the high-resolution solid-state γ detector has made it possible to follow the rotational bands up to as high as spin 18 (Stephens, Lark, Diamond)[1]. The new results showed that the energies of the high-spin rotational states deviate from the well-known rule of a rigid rotor, namely

$$E_{rot} = \frac{\hbar^2 I(I+1)}{2\mathcal{I}} \tag{1.1}$$

where $\mathcal{I}$ is the effective moment of inertia and is independent of I, the nuclear spin. The trend, as revealed by the new measurements, is that $\mathcal{I}$ increases with the spin I of the level.

A number of efforts have since been directed at understanding the deviations from the rigid rotor formula. There are two general approaches. One involves a microscopic calculation usually based on the second-order cranking model first suggested by Inglis[2]. Corrections arise from taking into account the centrifugal stretching and the Coriolis-antipairing (CAP) effect first suggested by Mottelson and Valatin[3] (Udagawa and Sheline[4]; Bes, Landowne, and Mariscotti[5]; Krumlinde[6]; and Marshalek[7]). Others have tried to fit the energy levels by two-parameter formulas of various forms[8-14]. Most of them can be derived either from the fourth-order cranking model (Harris[9]) or from the centrifugal stretching model, namely,

$$E_{rot} = \tfrac{1}{2} C_t \left(t - t_o\right)^2 + \frac{I(I+1)}{2\mathcal{J}(t)}$$
$$\frac{\partial E}{\partial t} = 0, \qquad (1.2)$$

with an assumed specific relation between the moment of inertia and the general stretching variable t. Other two- or three-parameter formulations[15,16] are based on a Taylor series expansion, however, the convergence of these series is questionable.

In the work presented in this thesis we have followed a semi-empirical approach in which we express E_{rot} as

$$E_{rot} = \beta\omega^2 + \gamma\omega^3 + \delta\omega^4 , \qquad (1.3)$$

where ω is the nuclear angular velocity. The expectation value of the angular momentum, $\langle\psi|J_x|\psi\rangle$, is then obtained using Feynman's theorem[17] and ω is eliminated from the equations for E_{rot} and $\langle\psi|J_x|\psi\rangle$, giving a three-parameter expression for the rotational energy. We employed a least-squares fitting procedure to determine the best values of the parameters for each nucleus.

In Chapter 2 the nuclear theory related to the rotation problem is discussed. The various efforts that have been made in understanding the deviations from the rigid rotor formula (1.1) are reviewed in Chapter 3. In Chapter 4 we propose a three-parameter semi-empirical model for the description of ground-state rotational bands and compare it to other models. It is found that our model gives excellent results for the level energies, and is characterized by a parameter which serves to modify the nuclear softness. Thus

negative nuclear softness may be obtained without requiring a negative ground state moment of inertia.

2. COLLECTIVE MODEL OF THE NUCLEUS

2.1 Introduction

The shell model of the nucleus considers the motions of individual nucleons in an isotropic average nuclear field generated by all the other nucleons. In many nuclei the existing combinations of nuclear orbitals give rise to an overall spherical distribution of nuclear matter, but in others this may not be so. In the latter case the average field will not be given by an isotropic average nuclear potential. In addition, the co-operative motion of many nucleons may result in collective oscillations of the nucleus as a whole about some equilibrium shape, which will play an essential role in the low energy spectra of the nucleus.

The shell model, in its simple form, ignores the important nucleon interactions which are not contained in the average field. If the nucleus had the structure implied by the concept of a liquid drop, it would always have its lowest energy for a spherical shape. But the individual nucleons have the characteristic properties given by the shell model which implies a systematic tendency for distortion of the nuclear shape. The basic mechanism for this lies in the effect first pointed out by Rainwater[18]. A single nucleon moving within the nucleus exerts a centrifugal pressure against the walls of the nucleus in its orbital plane and tends to produce an oblate deformation of the nuclear surface. When the nucleus has a closed shell configuration the deforming effects of many nucleons cancel out because the orbitals are oriented equally in all directions.

Where there are particles not in filled shells the tendency is for the nucleus to adjust its surface to coincide with the density distribution of these particles. If there were no opposing forces, this centrifugal pressure would result in a nucleus with a space distribution equivalent to that of the nucleons in the unfilled shells. There are, however, two effects working in the opposite direction. One is the difficulty in polarizing the closed shell core which strongly prefers spherical symmetry. The other is the pairing forces of the extra-core nucleons. When a nucleus has only a few nucleons beyond a closed shell these effects over-balance the distorting effects of the last odd nucleon and the nuclear equilibrium shape remains spherical. However, the nucleus does become less resistant to shape changes. This softness is evident in the decrease in the energy involved in collective vibrations about the spherical equilibrium shape.

When sufficiently many particles are added outside closed shells the spherical shape becomes unstable and the nucleus assumes a spheroidal equilibrium shape. When this occurs the collective motions of the nucleus will be of two types: rotational and vibrational.

It is possible to approximate these effects by replacing the spherically symmetric binding potential of the simple shell model with an adjustable anisotropic binding potential. There remain, however, some significant residual interactions which arise from the pairing forces between the nucleons outside the closed shells.

These tend to couple two equivalent nucleons to a state of zero angular momentum and thus counteract the tendency of the individual nucleons to deform the nuclear shape.

The most important collective degrees of freedom for the low energy nuclear properties are expected to be those associated with oscillations in shape with approximate preservation of the nuclear volume. The nuclear shape can be expressed in spherical harmonics as follows:

$$R(\theta,\varphi) = R_0\left\{1 + \sum_{\lambda\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\theta,\varphi)\right\} \qquad (2.1)$$

where R_0 is the equilibrium radius and $Y_{\lambda\mu}$ is the normalized spherical harmonic of order $\lambda\mu$. If we make the assumption that the constants $\alpha_{\lambda\mu}$ are small and that the frequencies of the single particle excitations are much greater than those involved in collective motions, we can separate the total wavefunction into a part describing the particle motion and a part describing the collective motion. An approximate expression for the Hamiltonian specifying the collective motion is of the form

$$H_{coll} = V_n(\alpha) + T_n(\dot{\alpha}) \quad . \qquad (2.2)$$

Here $V_n(\alpha)$ refers to the potential energy of the nucleus as a function of the shape defined by the coefficents α. The subscript, n, refers to the group of quantum numbers specifying the motions of all the particles in a nucleus with a shape defined by the α. The second term gives the kinetic energy involved in small oscillations of the nuclear shape. The predominant term is quadratic in the $\dot{\alpha}_{\lambda\mu}$ and the normal modes of vibration are of the harmonic oscillator type. In general, the oscillations in shape of the lowest order, $\lambda = 2$, are of

primary importance.

The variation of the $V_n(\alpha)$ with nuclear shape is determined by the number of particles outside the closed shell, the particular orbits which they fill, and their residual interactions. For a nucleus at a closed shell the interparticle forces of the core nucleons result in a strong preference for spherical symmetry and shape changes are firmly resisted. When a few additional particles are added there is, as we have seen, a competition between nuclear polarization and pairing forces in increasing and decreasing nuclear stiffness towards shape changes. As more and more particles are added beyond the closed-shell configuration the coherent effects of many particles ultimately bring about a stabilized deformation of the nucleus in which a potential energy minimum exists for a non-spherical shape. A stabilized non-spherical shape can be considered to be achieved when the shape changes associated with zero-point vibrations are small compared to the equilibrium deformation.

For non-spherical nuclei, the collective excitations include not only vibrational oscillations but changes in orientation without change in shape — that is, rotational excitation.

2.2 β - and γ - Vibrations

In the idealized case of a nucleus with constant density and a sharp surface, the nuclear surface would be defined by the $\alpha_{\lambda\mu}$ in the equation

$$R(\theta,\varphi) = R_0\left\{1 + \sum_{\lambda\mu} \alpha_{\lambda\mu}\, Y_{\lambda\mu}(\theta,\varphi)\right\} \qquad (2.3)$$

Here μ is the projection of λ on a space fixed axis. For small amplitudes of oscillations the energy may be expanded in powers of $\alpha_{\lambda\mu}$ and $\dot{\alpha}_{\lambda\mu}$ and one obtains, to a first approximation,

$$H_{coll} = \sum_{\lambda\mu}\left(\tfrac{1}{2} B_\lambda \left|\dot{\alpha}_{\lambda\mu}\right|^2 + \tfrac{1}{2} C_\lambda \left|\alpha_{\lambda\mu}\right|^2\right) \qquad (2.4)$$

corresponding to a set of independent harmonic oscillators with energy quanta

$$\hbar\omega_\lambda = \hbar\sqrt{\frac{C_\lambda}{B_\lambda}} \qquad (2.5)$$

The B_λ represents the mass transport associated with the vibration. A theoretical estimate based on the surface oscillations of an irrotational and incompressible liquid drop would give

$$(B_\lambda)_{irrot} = \frac{1}{\lambda}\,\frac{3}{4\pi} A M R_0^2 \qquad (2.6)$$

where AM is the mass of the nucleus. The parameter C_λ represents an effective surface tension.

The shape oscillations may be classified according to their multipole order λ. The lowest frequencies are expected to be of the quadrupole type ($\lambda = 2$) since a surface deformation with $\lambda = 1$ corresponds to a simple translational movement.

Let us now consider in more detail the possible types of surface vibration (of the quadrupole type). Choosing the axes of our co-ordinate system to coincide with the principle axes of an ellipsoidal nucleus simplifies the coefficents $\alpha_{\lambda\mu}$ as follows:

$$\alpha_{21} = \alpha_{2,-1} = 0$$
$$\alpha_{22} = \alpha_{2,-2} \,.$$

Also,

$$\alpha_{20} = \beta \cos\gamma$$

and

$$\alpha_{22} = \frac{1}{\sqrt{2}} \beta \sin\gamma$$

where β is a measure of the total deformation from a sphere and describes the deviation from rotational symmetry about the principal axis of the ellipsoid. It can be shown that the expansion of (2.3) under these conditions gives

$$R = R_0 \left\{ 1 + \beta \cos\gamma \sqrt{\frac{5}{16\pi}} (3\cos^2\theta - 1) + \frac{1}{\sqrt{2}} \beta \sin\gamma \sqrt{\frac{15}{32\pi}} \sin^2\theta \cos 2\varphi \right\}. \quad (2.7)$$

If $\gamma = 0^\circ$ the last term drops out. In this case, if $\beta > 0$, the nuclear shape is a prolate spheroid; if $\beta < 0$, it is an oblate spheroid. The term β - vibration refers to an oscillation in the shape parameter β. If $\gamma \neq 0^\circ$ the circular cross section of the nucleus perpendicular to the main axis is changed into an ellipse. The term γ - vibration may refer to oscillations around a spherical equilibrium shape, about a spheroidal equilibrium shape, or about an ellipsoidal equilibrium shape. The common feature of these three types is that the nuclear shape is ellipsoidal at the extremes of the oscillatory motion.

2.3 Rotational Energies and the Moment of Inertia

At a considerable distance from closed shells the nucleus becomes stabilized in a non-spherical shape under the influence of the coherent effects of many particles in unfilled shells. It should be noted that it is necessary to be far removed from closed shells

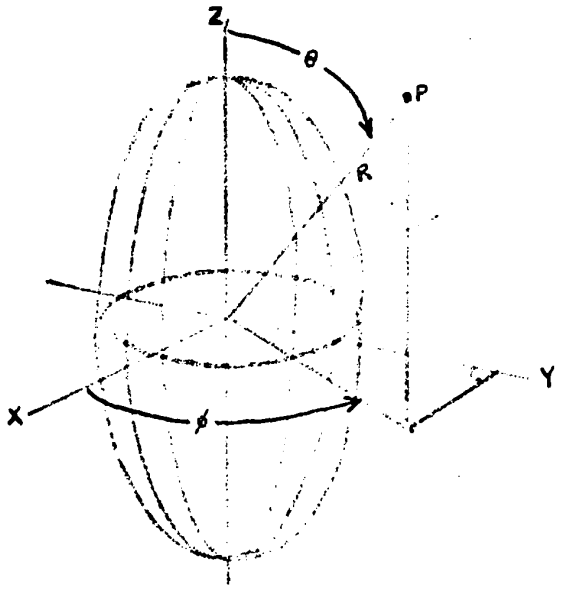

Figure 1. Shape of the nucleus given by Equation (2.7) for deformation with β positive and $\gamma = 0$.

for both neutrons and protons if conditions are to be favourable for stabilization of the nucleus in a spheroidal shape. Such a location is found among the heavy elements above Pb^{208} and among the heavy rare earths between the mass numbers 150 to 195. In these regions the nucleus acquires a prolate spheroidal shape[20].

For the strongly deformed nucleus the frequencies of motion which have to do with vibrational, and particularly with rotational, excitation are lower in general than those related to intrinsic particle excitation. In this case one can obtain an approximate separation of the motion of the individual particles in the potential field defined by the equilibrium shape of the core and the relatively slow collective rotation and vibration of the entire system. In other words, the complete wavefunction of the nucleus may be put as

$$\psi = \chi_{PART} \; \varphi_{VIB} \; D_{ROT} \; . \qquad (2.8)$$

Here χ_{part} represents the intrinsic motion of the nucleons which can be expressed in terms of the independent motion of the individual particles in the deformed field. φ_{vib} describes the vibrations of the nucleus around the equilibrium shape, and D_{rot} describes the collective rotational motion of the system as a whole.

The coupling scheme for deformed nuclei is illustrated in Figure 2. The three important constants of motion are $\bar{I}$, K, and M where $\bar{I}$ ($\equiv \hbar\, I(I+1)$)is the total angular momentum of the nucleus with component along the space axis $M\hbar$ and component along the symmetry axis $K\hbar$. Each individual particle of angular momentum $\bar{j}_i$ has a

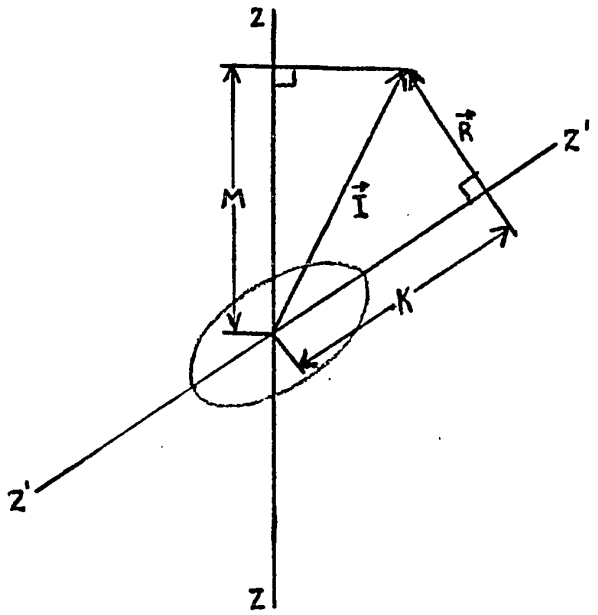

Figure 2. Coupling scheme appropriate for deformed nuclei. $\bar{I}$ is the total angular momentum of the nucleus. K and M are the projections of $\bar{I}$ on the axis of symmetry Z' and a space fixed axis Z, respectively. $\bar{R}$ is the collective rotational angular momentum. For the ground state and for many low-lying states K is equal to Ω where Ω is defined as the projection of the total angular momentum of the intrinsic particle motion on the symmetry axis.

projection on the symmetry axis Ω_i. Since $\sum_i \bar{j}_i = \bar{j}$ then $\sum_i \Omega_i = \Omega$. The angular momentum of the collective motion is $\bar{R}$. We thus obtain

$$|\bar{R}|^2 = \{I(I+1) - K^2\}\, \hbar^2 \quad . \tag{2.9}$$

For the ground state and for low-lying excited states for which there is no collective rotation about Z', $K = \Omega$. The total angular momentum $\sum_i \bar{j}_i$ of the particle system is not, in general, a constant of motion. The value of Ω for a single particle, Ω_p, takes on half-integral values, positive or negative. States differing only in the sign of Ω_p are degenerate since they are identical except for the opposite sense of rotation. The particles fill pairwise into states of opposite Ω_p with no net contribution to K from the pairs. Thus for an even-even nucleus in its lowest state, $K = 0$. (K will be different from Ω for γ - vibrations in which collective angular momentum is contributed along the nuclear axis.)

The deformed nucleus may rotate with preservation of shape and internal structure. Since $\bar{R}$ is always perpendicular to the symmetry axis, all members of a rotational band have the same value of K.

Provided we can separate the Hamiltonian as above we can express the nuclear rotational energy in a form similar to that of a free rotor. i.e.,

$$E_{rot} = \tfrac{1}{2} \mathcal{J} \omega^2 \tag{2.10}$$

where $\mathcal{J}$ is the effective moment of inertia about the axis of rotation perpendicular to the nuclear symmetry axis and ω is

the nuclear angular velocity. Also, since $\bar{R} = \mathcal{J}\omega$, we have, using (2.9),

$$E_{rot} = \frac{\hbar^2}{2\mathcal{J}} \{ I(I+1) - K^2 \} \quad . \tag{2.11}$$

Using this formula we see that the spacings should be

$$R_4 = E_{4+}/E_{2+} = 10/3 \ , \ R_6 = E_{6+}/E_{2+} = 21/3, \text{ etc.}.$$

($\hbar^2 K^2/2\mathcal{J}$ does not affect the level spacings.) The possible quantum states of the nucleus are restricted by the reflection symmetry of the deformation, which implies that the states labeled by (K, Ω) must be combined in a definite way with those labeled by $(-K, -\Omega)$. Also, the reflection symmetry about the plane passing through the center of the nucleus and perpendicular to the axis of symmetry implies that the collective motion has even parity and that the parity of a nuclear state is thus determined by that of the particle structure. This symmetry condition limits the acceptable states in the ground state rotational band of even-even nuclei to I = 0, 2, 2, ... even parity.

The collective motion which gives rise to the nuclear rotation is essentially different from that of a rigid body, and the effective moment of inertia, $\mathcal{J}$, is somewhat less than the rigid body moment, $\mathcal{J}_{rigid}$, given by

$$\mathcal{J}_{rigid} = \frac{2}{5} M A R_0^2 (1 + 0.31\beta + 0.44\beta^2 + \ldots). \tag{2.12}$$

In the early development of the collective model, estimates of were based on a hydrodynamical model of nuclear motion (to be discussed later). For the irrotational flow of an incompressible fluid, this model estimates the moment of inertia by the following equation:

$$\mathcal{J}_{\text{irrot}} = \frac{2}{5} A M R_o^2 \beta^2 \qquad (2.13)$$

where β is a deformation parameter of a spheroid given by

$$\beta = \frac{4}{3}\sqrt{\frac{\pi}{5}}\,\frac{\Delta R}{R_o} \qquad 1.06\,\frac{\Delta R}{R_o} \quad . \qquad (2.14)$$

Here, R_o is the mean nuclear radius and ΔR is the difference between the major and minor semi-axis of the spheroid. However, the values of $\mathcal{J}$ found experimentally are usually two or three times higher than $\mathcal{J}_{\text{irrot}}$ and much closer than predicted to $\mathcal{J}_{\text{rigid}}$. Hence, experimentally one observes that $3\,\mathcal{J}_{\text{irrot}} < \mathcal{J} < \frac{\mathcal{J}_{\text{rigid}}}{2}$.

3. PHENOMENOLOGICAL MODELS OF NUCLEAR ROTATION

The rotational formulas given in Chapter 2, based on the concept of a free rotor, can give precisely correct values for the spacing of rotational levels only if there is complete separation of rotational from vibrational and intrinsic motions. That this is not always the case is evident from the observed deviations in the spacings of the rotational levels from the $I(I + 1)$ law, which will predict a value of 10/3 for R_4 (the ratio of E_4 to E_2). It is observed that in many cases R_4 is less than 10/3. This leads to the following classification of R_4 regions:

I. Rotation Region: $3 \leq R_4 \leq 3.33$

II. Transition Region: $2.4 < R_4 < 3$

III. Vibration Region: $2 \leq R_4 \leq 2.4$.

It should be noted that $R_4 = 3.33$ corresponds to the ideal rigid rotor, while $R_4 = 2$ corresponds to the ideal quadrupole vibrator.

3.1 The Hydrodynamical Model

Many models have been introduced to describe this departure from the $I(I + 1)$ dependence. One of the first was the hydrodynamical model of Bohr and Mottelson[21]. They added an extra term proportional to $I^2(I + 1)^2$ to the expression (2.11) in order to take into account rotation-vibration coupling. Thus we have (K being zero for the ground state of even-even nuclei)

$$E_{rot} = A\, I(I + 1) + B\, I^2(I + 1)^2 \qquad (3.1)$$

where $A = h^2/2\mathcal{J}$ and B is the rotation-vibration coupling constant

similar to that found in molecular spectroscopy. The parameters, A and B, are determined from the experimental positions of the first two excited levels 2^+ and 4^+ in the ground state band. It has been generally believed that the experimental energy levels can be accurately reproduced by equation (3.1), although the second term represents only a first-order correction and higher-order terms should also be considered.

In the hydrodynamical model the parameter B may be related to the angular velocities of the β - and γ - vibrations through the relation

$$B_{theoret} = \frac{4A^3}{(\hbar\omega_\gamma)^2} + \frac{12A^3}{(\hbar\omega_\beta)^2} \qquad (3.2)$$

where ω_γ and ω_β are the angular velocities of the γ- and β - vibrations, respectively. However, it was found that the empirical values of B obtained from curve fitting were different by an order of magnitude[22] from those calculated from (3.2). It was soon found necessary to add the next order correction term, proportional to $I^3(I+1)^3$, to obtain a more satisfactory description of relatively high spin states[23]. Nathan and Nilsson[24] have included even the terms of higher orders thus giving an infinite series of the form

$$E_{rot} = A\,I(I+1) - B\,I^2(I+1)^2 + C\,I^3(I+1)^3 - \ldots . \qquad (3.3)$$

Discussions of possible approaches to this are given by Gupta and Sood[25].

3.2 The Asymmetric Roter Model

Since it was found that the power series expansion (3.3) in $I(I+1)$ was inadequate, several approaches have been made based on a more precise treatment of rotation-vibration coupling. Up to this point it was assumed that the shape of the nucleus in its ground state possesses axial symmetry. Davydov and his coworkers[26-28] have considered the consequences of dropping this assumption. First, Davydov and Filippov[26] considered the rotational states of a nucleus with an ellipsoidal shape under the adiabatic approximation that there was no interchange of energy with the intrinsic or vibrational states during rotation. The Hamiltonian for such a rotation is

$$H = \sum_{\lambda=1}^{3} \frac{A\,\bar{J}_\lambda^2}{2\,\sin^2\left(\gamma - \frac{2\pi}{3}\lambda\right)} \qquad (3.4)$$

where $A = \frac{\hbar^2}{4B\beta^2}$ and B is a mass parameter. The $\bar{J}_\lambda$ are operators of the projection of the nuclear angular momentum J on the axes of the co-ordinate system which are chosen to correspond to the principal axes of the ellipsoid. For $\gamma \neq 0$ or $\pi/3$, (3.4) represents the rotation of an asymmetric top. At $\gamma = 0$ we have a 2+, 4+, 6+ series of states which is identical in all respects to the ground state band discussed in Chapter 2 with the energy spacings following the $I(I+1)$ rule. As the deviations from axial symmetry increase, the level energies also increase slightly. (For example, the axial symmetry increases from $\gamma = 16^\circ$ to $\gamma = 25^\circ$ between Os^{186} and Os^{192}.) The minimum value for R_4 turns out to be 8/3. It is found that the parameter γ is simply related to $E_{2'}/E_2$. (Here $E_{2'}$ is defined as follows. There are found nuclei between the strongly deformed and "magic number" nuclei with a

near-harmonic pattern which is characterized by a second excited state with an energy approximately twice the energy of the first excited state and I = 0, 2, or 4. The second excited 2+ state is usually denoted by $2'+$ and its energy by $E_{2'}$.) A study of the branching ratios between the γ - vibrational (K = 2) and the ground state bands in the framework of the same model yields values of γ ranging from 12° to 23° for the same nuclei, suggesting a small inconsistency in the axially asymmetric model which is not easily removed[29].

The fact that most of the quantities predicted by the asymmetric rotor model are closely related to γ which is in turn related to $E_{2'}/E_2$ has encouraged a number of comparisons of experimental data as a function of this energy ratio. The most significant results are those of Mallmann[30] who showed that for even-even nuclei with widely differing N, Z, and E_2 values the energy ratios E_6/E_2 and E_8/E_2, plotted against E_4/E_2, lie on two "universal" curves. This finding suggests that the ground-state bands may indicate features which are common to nuclei lying both inside and outside the deformed region.

The asymmetric rotor model has been extended to include the effects of interacion rotational and vibrational modes by Davydov and Chaban[28]. They assume that the nucleus has an equilibrium value of β and of γ about which it can execute vibrations. The problem is to determine the coupling of these vibrations to the rotation as provided by the centrifugal forces. A parameter μ is introduced whose value serves as a guide to the extent of this coupling. With the aid of this parameter μ and the older parameter γ, Davydov and Chaban were

able to make many significant correlations of experimental spectra for a wide range of nuclei.

3.3 Models Based on Centrifugal Stretching

As was pointed out in Chapter 1, new methods for measuring energies showed that the level spacings at higher I are smaller than those given by the I(I + 1) rule. This decrease in the energy spacing may be attributed to an increase in the moment of inertia $\mathcal{J}$. At higher I the moment of inertia appears to approach a "rigid" value. Morinaga[31] introduced the parameter "softness", defined as the percentage increase of the moment of inertia per unit change of angular momentum, and discussed the form of dependence on this quantity on I as a function of N and Z.

3.3.1 The Semiclassical Model

One explanation for the increase of the moment of inertia as I increases is that at higher angular momenta the deformation (β) increases[32] (β stretching or centrifugal stretching). The semi-classical model of Diamond, Stephens, and Swaitecki[32], based on this assumption, leads to an expression for the energy as the sum of a potential energy term and a kinetic (rotational) energy term:

$$E_I(\beta) = \tfrac{1}{2} C (\beta_I - \beta_0)^2 + \{ I(I+1)/2\, \mathcal{J}(\beta_I) \} \quad (3.5)$$

where $\mathcal{J}$ is the moment of inertia in terms of $\hbar^2$. In addition, the equilibrium condition $\partial E_I / \partial \beta_I = 0$ is applied to obtain the value of β_I. With this model a good fit may be obtained for bands of

strongly deformed nuclei, assuming the relation given by the hydrodynamical model ($\mathcal{J} \sim \beta^2$). However, bands outside the deformed region cannot be fitted by this method with reasonable accuracy.[32]

In view of this, the deformation parameter β was replaced by a general stretching variable t in (3.5) and it was assumed that the moment of inertia could be expressed as $\mathcal{J} \approx t^n$; different values for n were chosen by different authors. Draper et al.[13] have taken n as a non-integer parameter to be determined separately for each nucleus. Their results indicate that the best n values range from 0.7 to 2.8 . Diamond et al.[32], Mosźkowski[10], and Sood[11] have used a quadratic dependence, $\mathcal{J} \approx t^2$, the same as that given by the irrotational flow model.

3.3.2 The Variable Moment of Inertia Model

Mariscotti et al.[12] have assumed a linear dependence $\mathcal{J} \approx t$ as an empirical fact based on the study[33] of the general relation $\mathcal{J} \approx t^n$. This is equivalent to taking the moment of inertia, $\mathcal{J}$, itself as the general variable t. They then arrive at the variable moment of inertia model (VMI).

The level energy is thus given by

$$E_I(\mathcal{J}) = \tfrac{1}{2} C (\mathcal{J}_I - \mathcal{J}_0)^2 + \tfrac{1}{2} \left[I(I+1)/\mathcal{J}_I \right] \quad (3.6)$$

subject to the equilibrium condition

$$\partial E(\mathcal{J})/\partial \mathcal{J} = 0. \quad (3.7)$$

The relation (3.7) determines the moment of inertia $\mathcal{J}_I$ (given in

units of $\hbar^2$) for each state with spin I. $\mathcal{J}_0$ is a parameter defined as the "ground state moment of inertia" and C (> 0) is the "restoring force constant"[33,34]. The model is successful in justifying Mallmann's curves, in going beyond the range of validity of the asymmetric rotor model, in predicting the levels of ground-state bands (they present data showing the predictions of the VMI model for the energy levels of 88 even-even nuclei), and in fitting the rotational bands built on γ - vibrational states in even-even nuclei.

For each spin I there exists an equilibrium value of the variable $\mathcal{J}$ determined from (3.7). Using both (3.6) and (3.7), a cubic in $\mathcal{J}$ is obtained which has one real root for any finite value of $\mathcal{J}_0$ and C.

$$\mathcal{J}_I^3 - \mathcal{J}_I^2 \mathcal{J}_0 - [I(I+1)/2C] = 0 . \tag{3.8}$$

Equation (3.8) combined with (3.6) yields the following expression for the energy of the state with spin I:

$$E_I = \frac{I(I+1)}{2\mathcal{J}_I}\left\{1 + \frac{I(I+1)}{4C\mathcal{J}_I^3}\right\} \tag{3.9}$$

From (3.8) the "softness"[31] of the nucleus turns out to be

$$\sigma = \left[\frac{1}{\mathcal{J}}\frac{d\mathcal{J}}{dI}\right]_{I=0} = \frac{1}{2C\mathcal{J}_0^3} . \tag{3.10}$$

The quantity σ, which may be taken to be a parameter, provides a convenient mode of discussing the range of validity of the VMI model. Defining $r_I = \mathcal{J}_I/\mathcal{J}_0$, we have from (3.8)

$$r_I^3 - r_I^2 = \sigma I(I+1) \tag{3.11}$$

In the adiabatic limit (rigid rotor), $\sigma = 0$ and hence $r_I = 1$.

Thus,

$$E_I(\sigma = 0) = I(I+1)/2\mathcal{I}_0 \tag{3.12}$$

In this limit, $R_I = E_I/E_2$ is

$$R_I(\sigma = 0) = \frac{I(I+1)}{6} \tag{3.13}$$

In the opposite extreme in the limit of very soft nuclei $\sigma \longrightarrow \infty$ and from (3.8) we have

$$r_I = \left\{\sigma\, I(I+1)\right\}^{1/3} \tag{3.14}$$

We then have

$$E_I(\sigma\to\infty) = \frac{3}{4}\left\{\frac{I(I+1)}{\mathcal{I}_1}\right\} \tag{3.15}$$

which leads to

$$R_I(\sigma\to\infty) = \left\{\frac{I(I+1)}{6}\right\}^{2/3} . \tag{3.16}$$

Hence the range of validity of the VMI model is

$$\left\{\frac{I(I+1)}{6}\right\}^{2/3} \leq R_I \leq \frac{I(I+1)}{6} . \tag{3.17}$$

In the case of $I = 4$ this gives

$$2.23 \leq R_4 \leq 3.33 \tag{3.18}$$

which is greater than that of the Davydov-Filippov model where $2.67 < R_4 < 3.33$ (see Sec 3.2).

G. Scharff-Goldhaber and A.S. Goldhaber[35] further extended the VMI model toward the magic nuclei by permitting negative values of the parameter $\mathcal{I}_0$. For large negative values of $\mathcal{I}_0$ they obtain $R_4 \to \sqrt{20/6} = 1.82$, which leads them to the following classification.

$2.23 < R_4 < 3.33$:	Deformed Region
$1.82 < R_4 < 2.23$:	Spherical Region
$1.00 < R_4 < 1.82$:	Magic Region

In the deformed region the ground state moment of inertia $\mathcal{I}_0$ is

positive, while in the spherical region it is negative. They introduce the notion of "internal stress" or "rigidity" to interpret the significance of $\mathcal{J}_0 < 0$. The larger the negative value of $\mathcal{J}$, the more firmly the shell structure resists departure from spherical symmetry. The fact that $\mathcal{J}_0$ changes sign at $R_4 = 2.23$ while C remains positive, means that σ becomes negative. This is not mentioned at all by Schraff-Goldhaber et al.[35].

Mariscotti[36], who also discusses the extended VMI model, develops three solutions for the rotational energy. For two of these solutions σ goes from a value of zero at $R_4 = 3.33$ to $+\infty$ at $R_4 = 2.23$. Then there is a discontinuity in σ as it jumps to $-\infty$ and then back to zero at $R_4 = 1.825$. The third solution gives the same σ down to $R_4 = 2.23$ but for $R_4 < 2.23$ it remains small and negative ($\approx 10^{-4}$). Mariscotti simply states that for $R_4 > 2.23$, $C/\mathcal{J}_0^3$ represents the softness, thus avoiding an interpretation of negative σ.

3.4 The Cranking Model

The second main approach to explain the increase in angular momentum with spin is the cranking model as developed by Inglis[2]. If the rotational motion is slow compared to the intrinsic motion (adiabatic condition), we can consider the energy required to rotate the system with its internal motion unchanged. Formally we transform the intrinsic Hamiltonian to a rotating co-ordinate system, which introduces a new term, $\hbar J_x \omega$, if the angular velocity of the system

is ω about the x axis and $\hbar \bar{J}$ is the total angular momentum. The change in energy due to this term is the rotational energy and the moment of inertia, $\mathcal{J}$, is twice the coefficient of ω^2. If second-order adiabatic perturbation theory is used one obtains

$$\mathcal{J} = 2\hbar^2 \sum_{i \neq 0} \frac{|\langle i|J_x|0\rangle|^2}{E_i - E_0} \tag{3.19}$$

where the label i runs over all the intrinsic states of the system, and E_i is the total energy of the state i.

If, for the intrinsic states, one substitutes into (3.19) shell-model states of individual particles moving in orbits appropriate to a deformed well, one obtains the rigid body moment of inertia[37]. This led Harris[9] (HPA) to extend the cranking model by including higher order terms in ω. He begins by considering a rotating, deformed, self-consistent potential well which, in the laboratory system, has a time-dependent Hamiltonian H and state function Ψ which is a solution of the equation

$$H\Psi = i\,\partial\Psi/\partial t \tag{3.20}$$

Assuming that the nucleus is rotating about the x axis, (3.20) is transformed to the intrinsic nuclear reference frame in which the wave function is φ where

$$\Psi = U(t)\,\varphi \tag{3.21}$$

Substituting (3.21) into (3.20) gives a new Schrodinger equation

$$\tilde{H}\,\varphi = i\,(\partial\varphi/\partial t) \tag{3.22}$$

where $\tilde{H}$ is given by

$$\tilde{H} = U^{-1}\,(HU - i\,\partial U/\partial t) \tag{3.23}$$

A stationary solution for (3.21) for which one can write

$$\tilde{H}\,\tilde{\psi} = \tilde{E}\,\tilde{\psi} \tag{3.24}$$

Since U is given by

$$U = \exp(-iJ_x\omega t) \tag{3.25}$$

equation (3.23) may be written as

$$\begin{aligned}\tilde{H} &= \exp(iJ_x\omega t)\,H\,\exp(-iJ_x\omega t) - \omega J_x \\ &= H_0 - \omega J_x = H_0 - H'\end{aligned} \tag{3.26}$$

The energy eigenvalues in the two systems are also easily related;

$$\begin{aligned}E &= \langle\psi|H|\psi\rangle \\ &= \tilde{E} + \omega\langle\psi|J_x|\psi\rangle .\end{aligned} \tag{3.27}$$

Equation (3.24) is then solved using (3.26) with basis states taken as stationary states of H_0, and with H' treated as a perturbation. The usual cranking model (Inglis) results from second-order perturbation theory. Harris includes terms up to fourth order in H'. The result is

$$\begin{aligned}E = E_0 &- \omega^2 \sum_m{}' \frac{\langle 0|J_x|m\rangle\langle m|J_x|0\rangle}{E_0 - E_m} \\ &- 3\omega^4 \sum_{mnp}{}' \frac{\langle 0|J_x|m\rangle\langle m|J_x|n\rangle\langle n|J_x|p\rangle\langle p|J_x|0\rangle}{(E_0-E_m)(E_0-E_n)(E_0-E_p)} \\ &+ 3\omega^4 \sum_{mn}{}' \frac{|\langle 0|J_x|n\rangle|^2\,|\langle 0|J_x|m\rangle|^2}{(E_0-E_m)(E_0-E_n)^2}\end{aligned} \tag{3.28}$$

All terms containing ω contribute to the rotational energy. Equation (3.28) may be written as

$$E = E_0 + \tfrac{1}{2}\mathcal{J}(\omega)\,\omega^2 \tag{3.29}$$

where

$$\mathcal{J}(\omega) = \mathcal{J}_0 + 3C\omega^2 \tag{3.30}$$

and

$$\mathcal{J}_0 = 2\sum_m{}' \frac{|\langle m|J_x|0\rangle|^2}{E_m - E_0} \tag{3.31}$$

with

$$\begin{aligned}C = 2\sum_{mnp}{}' &\frac{\langle 0|J_x|m\rangle\langle m|J_x|n\rangle\langle n|J_x|p\rangle\langle p|J_x|0\rangle}{(E_m-E_0)(E_n-E_0)(E_p-E_0)} \\ &- \mathcal{J}_0 \sum_m{}' \frac{|\langle m|J_x|0\rangle|^2}{(E_m-E_0)^2}\end{aligned} \tag{3.34}$$

Also one obtains

$$\langle \varphi | J_x | \varphi \rangle = \omega \left(\mathcal{J}_0 + 2C\omega^2 \right). \tag{3.33}$$

It is evident that a different effective moment of inertia enters into the calculation of energy and angular momentum when higher-order terms are retained in $\mathcal{J}(\omega)$. The usual cranking model treatment leads to (3.30), but here one has a result for $\mathcal{J}$ which depends on the degree of rotation. This is similar to the theory of Davydov and Chaban although no interaction between rotational and vibrational modes is explicitly introduced.

Harris proceeds to obtain the main results of the above without resorting to perturbation theory, by employing a "self-consistency" argument via Feynman's theorem[17] (see Appendix A). The results are

$$E_{rot} = \tfrac{1}{2}\omega^2 \left(\mathcal{J}_0 + 3C\omega^2 + 5D\omega^4 + \ldots \right) \tag{3.34}$$

$$\left\{ I(I+1) \right\}^{1/2} = \omega \left(\mathcal{J}_0 + 2C\omega^2 + 3D\omega^4 + \ldots \right) \tag{3.35}$$

where $\mathcal{J}_0$, C, D, ... are adjustable parameters. In principle, ω may be eliminated from these two equations leaving one equation for E_{rot} as a function of nuclear spin I. Harris presents energies for even-even nuclei obtained from empirical curve fittings to the experimentally observed levels in which he has retained two and three parameters, respectively, in the equations (3.34, 3.35).

It may be shown that the Harris model and the VMI model are equivalent. The two-parameter Harris model reduces to

$$\left\{ I(I+1) \right\}^{1/2} = \omega \left(\mathcal{J}_0' + 2C'\omega^2 \right) \tag{3.36}$$

$$E'_I = \tfrac{1}{2}\omega^2 (\mathcal{J}'_o + 3C'\omega^2) . \qquad (3.37)$$

If the moment of inertia $\mathcal{J}_I$ is defined as

$$\mathcal{J}_I = \frac{\{I(I+1)\}^{1/2}}{\omega} \qquad (3.38)$$

one obtains from (3.36)

$$\mathcal{J}_I = \mathcal{J}'_o + 2C'\omega^2$$

$$= \mathcal{J}'_o + 2C' \frac{I(I+1)}{\mathcal{J}_I^2} \qquad (3.39)$$

or, equivalently,

$$\mathcal{J}_I^3 - \mathcal{J}_I^2 \mathcal{J}'_o - 2C' I(I+1) = 0 \qquad (3.40)$$

which is identical to (3.8) if

$$C' = 1/4C \quad \text{and} \quad \mathcal{J}'_o = \mathcal{J}_o . \qquad (3.41)$$

Using (3.39, 3.41), one may write (3.37) as

$$E'_I = \tfrac{1}{2}\omega^2 (\mathcal{J}_I + C'\omega^2).$$

Substituting from (3.38) one obtains

$$E'_I = \frac{I(I+1)}{2\mathcal{J}_I} \left\{ 1 + C' \frac{I(I+1)}{\mathcal{J}_I^3} \right\} \qquad (3.42)$$

Using (3.41) one finds that (3.42) is identical to (3.9).

3.4 Models Employing Taylor Series Expansions

Gupta[15] has developed a nuclear-softness (NS) model in which he treats the moment of inertia $\mathcal{J}_I$ as a function of spin I and expands $\mathcal{J}_I$ about its ground state value $\mathcal{J}_o$ for I = 0. This leads to

$$E_I = \frac{\hbar^2}{2\mathcal{J}_I} I(I+1)$$

$$= \frac{\hbar^2}{2}\left[\frac{1}{\mathcal{J}_0} - \left(\frac{1}{\mathcal{J}_I^2}\frac{\partial \mathcal{J}_I}{\partial I}\right)_{I=0} I + \left(\frac{2}{\mathcal{J}_I^3}\left(\frac{\partial \mathcal{J}_I}{\partial I}\right)^2 - \frac{1}{\mathcal{J}_I^2}\frac{\partial^2 \mathcal{J}_I}{\partial I^2}\right)_{I=0}\frac{I^2}{2!} + \cdots\right] \quad (3.43)$$

$$\times\ I(I+1)$$

Collecting the various terms in this expansion in terms of Morinaga's softness parameter[31], he obtains

$$E_I = \frac{\hbar^2 I(I+1)}{2\mathcal{J}_0}\ \frac{1}{1+\sigma_1 I}\left(1 - \frac{\sigma_2 I^2}{1+\sigma_1 I+\sigma_2 I^2} - \frac{\sigma_3 I^3}{1+\sigma_1 I+\sigma_2 I^2} + \cdots\right) \quad (3.44)$$

where

$$\sigma_1 = \frac{1}{\mathcal{J}_0}\frac{\partial \mathcal{J}_0}{\partial I}\ ,\ \sigma_2 = \frac{1}{2!\,\mathcal{J}_0}\frac{\partial^2 \mathcal{J}_0}{\partial I^2}\ ,\ \sigma_3 = \frac{1}{3!}\frac{\partial^3 \mathcal{J}_0}{\partial I^3}\ ,\ \cdots\ . \quad (3.45)$$

Treating $\mathcal{J}_0$, σ_1, σ_2, ... as adjustable parameters allows him to obtain a least-squares fit to the experimental data. Results for two- and three-parameter fits are given for 102 even-even nuclei. His results for two parameters are comparable to those obtained by VMI. Gupta did not discuss the convergence of his series nor the relative magnitudes of his terms.

Satpathy and Satpathy[16] have formulated a shape fluctuation (SF) model in which they write

$$E_I = E(\varphi(I)) + B(\varphi(I)\ I(I+1) \quad . \quad (3.46)$$

They then expand $\varphi(I)$ in a Taylor series about $I = 0$ and further expand E and B about $\varphi_{I=0}$ to obtain

$$E_I = E(\varphi_{I=0}) + I\left(\frac{\partial \varphi}{\partial I}\right)_{I=0}\left(\frac{\partial E}{\partial \varphi}\right)_{\varphi_{I=0}} + \cdots$$

$$+ \left\{B(\varphi_{I=0}) + I\left(\frac{\partial \varphi}{\partial I}\right)_{I=0}\left(\frac{\partial B}{\partial \varphi}\right)_{\varphi_{I=0}}\right\}\ I(I+1)$$

$$\approx E_0 + I\varphi' E' + (B_0 + I\varphi' B')\ I(I+1)$$

$$= E_o + B_o I(I+1) + I\,\varphi' E' + I\,\varphi' B' \quad I(I+1)$$
$$= aI + bI^2 + CI^3 \qquad (3.47)$$

where the three adjustable parameters a, b, and c are given by

$$a = B_o + \varphi' E'$$
$$b = B_o + \varphi' B' \qquad (3.48)$$
$$c = \varphi' B'$$

The three-parameter fit they obtain is comparable to Gupta's three-parameter NS model. Satpathy et al. also do not discuss the convergence of their series or the relative magnitude of various terms.

3.5 Expansions in Powers of Nuclear Spin

As was indicated in Sec. 3.1, for the region of strongly deformed nuclei there is sanction for writing the rotational energy as a power series in $I(I+1)$,

$$E_I = I(I+1)/2\mathcal{J} - B\,I^2(I+1)^2 + \ldots (3.49)$$

For the most strongly deformed nuclei we have $2\mathcal{J}B \sim 10^{-3}$. Therefore for $I = 10$, the second term is only 10% of the first. By contrast, we find that at the end of the deformed region $2\mathcal{J}B \sim 10^{-2}$. Thus mathematical convergence fails at $I \sim 10$. To deal with this problem Das, Dreizler, and Klein[38] have proposed an anharmonic vibration model (AVM) in which they expand the excitation energy as a polynomial in I rather than in $I(I+1)$. Their energy can be written as

$$E_I = a\,I + k\,I(I+1) \qquad (3.50)$$

where a and k are adjustable parameters. Their results are better than the VMI model in the vibration region but not as good in the rotation region.

In the vibrational region and in the beginning of the transition region, in the model of Das et al.[37], the first term dominates, while the second and third are in competition. In the rotation region and the end of the transition region their first two terms are in competition but their third term is definitely smaller. They claim that their analysis of the HPA and the VMI model yields the facts that the power series in ω is dominated by one term in the rotation region but that the first two terms compete in the vibration region. They conclude that to obtain a precision fit in the vibration region requires a formulation with more than two parameters. It is interesting to note that Ejiri[39] arrived at (3.50) on purely empirical grounds.

Marshalek[40] has developed a microscopic cranking model for vibrational nuclei. He claims, as a result of a microscopic calculation, that, for spherical nuclei (vibrational region), an expansion in I rather than in I(I + 1) is more likely to converge. Truncation of his expression to accomodate two parameters also yields (3.50).

4. THREE PARAMETER SEMI-EMPIRICAL APPROACH

4.1 General Solution and Parameters

We choose to write E in the laboratory frame as a polynomial in ω up to ω^4 :

$$E = E_o + \alpha\omega + \beta\omega^2 + \gamma\omega^3 + \delta\omega^4 \quad (\omega \geq 0) \tag{4.1}$$

where

$$\left.\begin{aligned} \alpha &= \left(\frac{\partial E}{\partial \omega}\right)_{\omega=0+} \\ \beta &= \frac{1}{2}\left(\frac{\partial^2 E}{\partial \omega^2}\right)_{\omega=0+} \\ \gamma &= \frac{1}{3!}\left(\frac{\partial^3 E}{\partial \omega^3}\right)_{\omega=0+} \\ \delta &= \frac{1}{4!}\left(\frac{\partial^4 E}{\partial \omega^4}\right)_{\omega=0+} \end{aligned}\right\} \tag{4.2}$$

For time-reversal invariance, we impose the condition on E that

$$E(\omega) = E(-\omega) \tag{4.3}$$

which implies that

$$\left(\frac{\partial^n E}{\partial \omega^n}\right)_{\omega=0+} = (-1)^n \left(\frac{\partial^n E}{\partial \omega^n}\right)_{\omega=0-} . \tag{4.4}$$

Also, analyticity of E at $\omega = 0$ requires that

$$\left(\frac{\partial E}{\partial \omega}\right)_{\omega=0+} = \left(\frac{\partial E}{\partial \omega}\right)_{\omega=0-} . \tag{4.5}$$

Relation (4.5) and (4.4) require that $\alpha = 0$ in (4.1).

For a description of the nucleus, consider a rotating, deformed, self-consistent potential well. In the laboratory system one has a time-dependent Hamiltonian H and a state-function ψ such that

$$H\psi = i\frac{\partial \psi}{\partial t} \tag{4.6}$$

We can transform (4.6) to the intrinsic nuclear reference frame. The wave function in the latter system is φ where

$$\psi = U(t)\,\varphi \,. \tag{4.7}$$

Substituting (4.7) into (4.6), one obtains a new Schrodinger equation

$$\tilde{H}\varphi = i\,\frac{\partial \varphi}{\partial t} \tag{4.8}$$

where

$$\tilde{H} = U^{-1}\left(HU - i\,\frac{\partial U}{\partial t}\right). \tag{4.9}$$

We now take a stationary solution of (4.8). i.e. one for which

$$\tilde{H}\,\varphi = \tilde{E}\,\varphi \,. \tag{4.10}$$

Noting that U is given by

$$U = \exp(-i\,J_x\,\omega t) \tag{4.11}$$

(assuming that the nucleus is rotating about the x axis) equation (4.9) may be written as

$$\begin{aligned}\tilde{H} &= \exp(i\,J_x\,\omega t)\,H\exp(-i\,J_x\,\omega t) - \omega J_x \\ &= H_o - \omega J_x \\ &= H_o - H' \end{aligned} \tag{4.12}$$

The energy eigenvalues are related by

$$\begin{aligned}E &= \langle\varphi|H|\varphi\rangle \\ &= \tilde{E} + \omega\langle\varphi|J_x|\varphi\rangle \,. \end{aligned} \tag{4.13}$$

We now express the expectation value of the angular momentum as the polynomial

$$\langle\varphi|J_x|\varphi\rangle = \beta'\omega + \gamma'\omega^2 + \delta'\omega^3 \,. \tag{4.14}$$

Thus (4.13) and (4.14) give

$$E = \tilde{E} + \omega^2(\beta' + \gamma'\omega + \delta'\omega^2) \tag{4.15}$$

Therefore

$$\frac{\partial E}{\partial \omega} = \frac{\partial \tilde{E}}{\partial \omega} + 2\beta'\omega + 3\gamma'\omega^2 + 4\delta'\omega^3 . \qquad (4.16)$$

Applying now Feynman's theorem (see proof in Appendix A) which states that for a stationary solution of

$$\tilde{H}(\omega)\,\psi(\omega) = \tilde{E}(\omega)\,\psi(\omega) \qquad (4.17)$$

one has

$$\frac{\partial \tilde{E}}{\partial \omega} = \left\langle \psi \left| \frac{\partial \tilde{H}}{\partial \omega} \right| \psi \right\rangle , \qquad (4.18)$$

we obtain

$$\frac{\partial \tilde{E}}{\partial \omega} = -\langle \psi | J_x | \psi \rangle . \qquad (4.19)$$

From (4.14), (4.16), and (4.19) we have

$$\frac{\partial E}{\partial \omega} = \beta'\omega + 2\gamma'\omega^2 + 3\delta'\omega^3 . \qquad (4.20)$$

But (4.1) gives

$$\frac{\partial E}{\partial \omega} = \alpha + 2\beta\omega + 3\gamma\omega^2 + 4\delta\omega^3 . \qquad (4.21)$$

Equating coefficents of like terms in (4.20) and (4.21) gives

$$\begin{aligned} \alpha &= 0 \quad \text{(as required)} \\ \beta &= \beta'/2 \\ \gamma &= 2\gamma'/3 \\ \delta &= 3\delta'/4 \end{aligned} \qquad (4.22)$$

Putting

$$\beta' = \mathcal{J}_0 , \quad \frac{\gamma'}{3} = B, \quad \frac{\delta'}{4} = C \qquad (4.23)$$

in (4.1) and (4.14) gives, respectively

$$E = E_0 + \frac{\omega^2}{2}(\mathcal{J}_0 + 4B\omega + 3C\omega^2) \qquad (4.24A)$$

$$\langle \psi | J_x | \psi \rangle = \omega\,(\mathcal{J}_0 + 3B\omega + 2C\omega^2) \qquad (4.25A)$$

Since $\langle \psi | J_x | \psi \rangle = \sqrt{I(I+1)}$, where I is the spin of the band head, we may begin with

$$E_{rot} = \frac{\omega^2}{2}\left(\mathcal{J}_o + 4B\omega + 3C\omega^2\right) \quad (4.24B)$$

$$\sqrt{I(I+1)} = \omega\left(\mathcal{J}_o + 3B\omega + 2C\omega^2\right) \quad (4.25B)$$

The nuclear angular velocity, ω, may be eliminated from these equations, leaving one equation for E_{rot} as a function of nuclear spin I, dependent on the three parameters $\mathcal{J}_o$, B, and C. Solving equation (4.25B), a cubic in ω, gives three roots[41] ω, ω_1, and ω_2 where

$$\omega = \frac{-B}{2C} + \left\{\frac{-r}{2} + \sqrt{\frac{r^2}{4} + \frac{q^3}{27}}\right\}^{1/3} + \left\{\frac{-r}{2} - \sqrt{\frac{r^2}{4} + \frac{q^3}{27}}\right\}^{1/3}, \quad (4.26A)$$

$$\omega_1 = \frac{-B}{2C} + e^{2\pi i/3}\left\{\frac{-r}{2} + \sqrt{\frac{r^2}{4} + \frac{q^3}{27}}\right\}^{1/3} + e^{4\pi i/3}\left\{\frac{-r}{2} - \sqrt{\frac{r^2}{4} + \frac{q^3}{27}}\right\}^{1/3}, \quad (4.26B)$$

$$\omega_2 = \frac{-B}{2C} + e^{4\pi i/3}\left\{\frac{-r}{2} + \sqrt{\frac{r^2}{4} + \frac{q^3}{27}}\right\}^{1/3} + e^{2\pi i/3}\left\{\frac{-r}{2} - \sqrt{\frac{r^2}{4} + \frac{q^3}{27}}\right\}^{1/3}, \quad (4.26C)$$

where

$$\left.\begin{aligned} r &= \frac{B^3}{4C^3} - \frac{\mathcal{J}_o B}{4C^2} - \frac{\sqrt{I(I+1)}}{2C} \\ q &= \frac{\mathcal{J}_o}{2C} - \frac{3B^2}{4C^2} \end{aligned}\right\} \quad (4.27)$$

From (4.25B) for I = 0 we have either

$$\omega = 0 \quad (4.28A)$$

or

$$\omega = \frac{-3B \pm \sqrt{9B^2 - 8\mathcal{J}_o C}}{4C} \quad (4.28B)$$

That $\omega = 0$ for I = 0 corresponds to the root (4.26A) can be

seen as follows. We note that the parameters $\mathcal{J}_o$, B, and C calculated by the computer in each case are such that one always has

$$9\,B^2 < 8\,\mathcal{J}_o\,C \tag{4.29}$$

The condition (4.29) implies that the roots (4.28B) are complex. Moreover, the condition, $9\,B^2 < 8\,\mathcal{J}_o\,C$, implies that

$$q > \frac{-\mathcal{J}_o}{6\,C} \tag{4.30}$$

which gives

$$\frac{r^2}{4} + \frac{q^3}{27} > 0 \quad . \tag{4.31}$$

From equation (4.31) we see that the root (4.26A) is real ($I > 0$) (the roots (4.26B) and (4.26C) are then complex). Therefore, for $I = 0$, the root (4.26A) gives $\omega = 0$.

We now consider some alternate expressions for E_{rot} which are of interest. First, we define the effective moment of inertia, $\mathcal{J}$, through the semi-classical relation

$$\sqrt{I(I+1)} = \omega\mathcal{J} = \omega(\mathcal{J}_o + 3B\omega + 2C\omega^2) \tag{4.32}$$

With this, we may write (4.24B) as

$$E_{rot} = \frac{\omega^2}{2}(\mathcal{J}_o + 4B\omega + 3C\omega^2)$$

$$= \frac{\omega^2}{2}(\mathcal{J} + B\omega + C\omega^2)$$

<u>i.e.</u>

$$E_{rot} = \frac{I(I+1)}{2\mathcal{J}} + \tfrac{1}{2}B\omega^3 + \tfrac{1}{2}C\omega^4 \tag{4.33}$$

Now consider

$$\frac{(\mathcal{J} - \mathcal{J}_o)^2}{8C} = \frac{9B^2\omega^2}{8C} + B\omega^3 + \left(\tfrac{1}{2}B\omega^3 + \tfrac{1}{2}C\omega^4\right). \tag{4.34}$$

Comparing (4.33) and (4.34) gives

$$E_{rot} = \frac{(\mathcal{J} - \mathcal{J}_o)^2}{8C} + \frac{I(I+1)}{2\mathcal{J}} - \frac{9B^2\omega^2}{8C} - B\omega^3$$

or, using (4.32)

$$E_{rot} = \frac{(\mathcal{J} - \mathcal{J}_o)^2}{8C} + \frac{I(I+1)}{2\mathcal{J}} - \frac{9B^2 I(I+1)}{8C\,\mathcal{J}^2} - \frac{B\{I(I+1)\}^{3/2}}{\mathcal{J}^3}. \quad (4.35)$$

This form of the expression for E_{rot} will prove useful in our discussion of nuclear softness. If B = 0, it is noted that (4.35) reduces to the VMI expression for E_{rot}.

Using (4.32), the expression for E_{rot} may also be written as

$$\begin{aligned} E_{rot} &= \tfrac{1}{2}\omega^2(\mathcal{J}_o + 4B\omega + 3C\omega^2) \\ &= \tfrac{1}{2}\omega^2(\mathcal{J} + B\omega + C\omega^2) \\ &= \frac{I(I+1)}{2\mathcal{J}}\left(\mathcal{J} + \frac{B\sqrt{I(I+1)}}{\mathcal{J}^2} + \frac{C\,I(I+1)}{\mathcal{J}^3}\right). \end{aligned}$$

<u>i.e.</u>

$$E_{rot} = \frac{I(I+1)}{2\mathcal{J}}\left(1 + \frac{B\sqrt{I(I+1)}}{\mathcal{J}^2} + \frac{C\,I(I+1)}{\mathcal{J}^3}\right). \quad (4.36)$$

This demonstrates that E_{rot} may be expressed as a polynomial in $\sqrt{I(I+1)}$. It is seen that (4.36) reduces to an alternate VMI expression for E_{rot} if B = 0.

Equation (4.36) could be used to provide a description in terms of the three parameters $\mathcal{J}_o$, B, and C with $\mathcal{J} = \mathcal{J}(\mathcal{J}_o, B, C)$, independent of ω. This form for E_{rot} suggests that one may write

$$E_{rot} = \sum_{n=2,3,\ldots} A_n \{I(I+1)\}^{n/2}. \quad (4.37)$$

This is quite plausible since, as has been noted, an expansion in I(I + 1) gives good results in the rotation region while an expansion in I gives good results in the vibration region. As will be shown,

(4.36), or, analagously,(4.37), gives an expansion in a quantity which is "in between" I and I(I + 1). Such an expansion will naturally give good fits in both regions.

We now investigate the possible physical significance of the three parameters appearing in the expression for E_{rot}. From (4.29) and (4.32) it is evident that $\mathcal{J}(I = 0) = \mathcal{J}_0$. Thus $\mathcal{J}_0$ is the "ground state moment of inertia". Noting that the expectation value of the nuclear angular momentum is given by $\langle \psi | J_x | \psi \rangle = \sqrt{I(I+1)}$ and that $\mathcal{J}$ is the rotational analogue of a mass we see that the term $\frac{I(I+1)}{2\mathcal{J}}$ in (4.35) represents the rotational kinetic energy. Similarly we may interpret the term (in (4.35)) $\frac{(\mathcal{J} - \mathcal{J}_0)^2}{8C}$ as representing the potential energy of the nucleus due to rotational stretching. The constant $1/4C$ appears as a "restoring force constant", in effect measuring the slope of the potential energy curve.

It remains to interpret the terms containing the parameter B. First, we demonstrate that the condition

$$\sqrt{I(I+1)} = \omega\mathcal{J} = \omega(\mathcal{J}_0 + 3B\omega + 2C\omega^2) \tag{4.32}$$

implies the equilibrium condition $\partial E/\partial \mathcal{J} = 0$. From equation (4.33) we get

$$\frac{\partial E}{\partial \omega} = -\frac{I(I+1)}{2\mathcal{J}^2}\frac{\partial \mathcal{J}}{\partial \omega} + \frac{3}{2}B\omega^2 + 2C\omega^2. \tag{4.38}$$

But from (4.32)

$$\frac{\partial \mathcal{J}}{\partial \omega} = 3B + 4C\omega. \tag{4.39}$$

Using now (4.39), we have for (4.38)

$$\frac{\partial E}{\partial \omega} = -\frac{I(I+1)}{2\mathcal{J}^2}(3B + 4C\omega) + \frac{3}{2}B\omega^2 + 2C\omega^3$$

$$= -\tfrac{1}{2}\omega^2(3B + 4C\omega) + \frac{3}{2}B\omega^2 + 2C\omega^3$$

$$= 0 \quad . \tag{4.40}$$

Hence

$$\frac{\partial E}{\partial \mathcal{J}} = 0 \qquad \text{for all } I \ , \tag{4.41}$$

since

$$\frac{\partial E}{\partial \mathcal{J}} = \frac{\partial E}{\partial \omega}\frac{\partial \omega}{\partial \mathcal{J}} \ . \tag{4.42}$$

This demonstrates that $\sqrt{I(I+1)} = \omega\mathcal{J}$ is equivalent to the equilibrium condition $\partial E/\partial \mathcal{J} = 0$. From (4.35) we have

$$\frac{\partial E}{\partial \mathcal{J}} = \frac{\mathcal{J} - \mathcal{J}_0}{4C} + \frac{9B^2}{4C}\frac{I(I+1)}{\mathcal{J}^3} + \frac{3B\{I(I+1)\}^{1/2}}{\mathcal{J}^4} - \frac{I(I+1)}{2\mathcal{J}^2} \ . \tag{4.43}$$

Using the equilibrium condition, (4.41) and rearranging we obtain

$$\mathcal{J}^5 - \mathcal{J}_0\mathcal{J}^4 + 9B^2\mathcal{J}I(I+1) + 12BC\{I(I+1)\}^{1/2} - 2C\mathcal{J}^2I(I+1) = 0 \tag{4.44}$$

From this equation $\mathcal{J}$ can be expressed as a function of the parameters $\mathcal{J}_0$, B, and C and of the nuclear spin I. We may now derive from (4.44) an expression for the nuclear softness[31], σ, defined as the relative increase of the moment of inertia with the angular momentum I.

Explicitly

$$\sigma \equiv \left[\frac{1}{\mathcal{J}}\frac{d\mathcal{J}}{dI}\right]_{I=0} = \frac{2\mathcal{J}_0C - 9B^2}{\mathcal{J}_0^4} \tag{4.45}$$

Noting that the VMI value for σ is $\sigma_{VMI} = 2C/\mathcal{J}_0^3$, (4.45) may be put as

$$\sigma = \sigma_{VMI} - \frac{9B^2}{\mathcal{J}_0^4} \ . \tag{4.46}$$

It is thus evident that the parameter B modifies the nuclear softness by reducing it somewhat below that given by the VMI model. A decrease in "softness" would lower the rotational energy levels since the nucleus is more resistent to deformation and thus would not acquire a large moment of inertia. This is also related to the fact that the terms dependent on B in (4.35) are negative, thus reducing the energy. This is obvious for $B > 0$, but for $B < 0$, the last term, $-B\{I(I+1)\}^{3/2}/\mathcal{J}^3$, would be positive but its effect would be much smaller, compared to that of the term $-\frac{9B^2}{8C}\frac{I(I+1)}{\mathcal{J}^2}$ which is negative regardless of the sign of B because of the larger power of $\mathcal{J}$ occuring in the denominator. (Note that the computer calculations indicate that $\mathcal{J} > B$.)

4.2 Range of Validity

We now determine the limits of validity of our semi-empirical approach. Defining $r = \mathcal{J}/\mathcal{J}_0$ we obtain from (4.44)

$$r^5 - r^4 + \frac{9B^2}{\mathcal{J}_0^4} r\, I(I+1) + \frac{12BC}{\mathcal{J}_0^5}\{I(I+1)\}^{3/2} - \frac{2C}{\mathcal{J}_0^3} r^2 I(I+1) = 0 \qquad (4.47)$$

This may be put in terms of σ as

$$r^5 - r^4 + \frac{9B^2}{\mathcal{J}_0^4} I(I+1)r(1-r) + \frac{12BC}{\mathcal{J}_0^5}\{I(I+1)\}^{3/2} = \sigma r^2 I(I+1) \qquad (4.48)$$

For the case of hard, well-deformed nuclei, $\sigma \to 0$ implying that $B \to 0$ and $C \to 0$ from equation (4.45). Hence (4.48) reduces to $r^5 - r^4 = 0$ or $r = 1$. That is $\mathcal{J} = \mathcal{J}_0$. Thus from (4.36)

$$E_{rot}(\sigma \to 0) = \frac{I(I+1)}{2\mathcal{J}_0}, \qquad (4.49)$$

which corresponds to the energy of a rigid rotor, and

$$R_4(\sigma \to 0) = \frac{10}{3} = 3.33 \quad , \tag{4.50}$$

which is the rigid rotor limit.

For the case of $|\sigma| \to \infty$, we have $|\mathcal{J}_0| \to 0$ and hence $|r| \to \infty$. Equation (4.48) can be put as

$$r^3 - r^2 + \frac{9B^2(1-r)}{r\,\mathcal{J}_0^4} I(I+1) + \frac{12BC\{I(I+1)\}^{3/2}}{r^2\,\mathcal{J}_0^5} = \sigma I(I+1) \tag{4.51}$$

Since B, C $\ll$ 1 and $|r| \to \infty$, this reduces to

$$r^3 \approx \sigma\, I(I+1)$$

or

$$r \approx \{\sigma I(I+1)\}^{1/3} . \tag{4.52}$$

Thus

$$\mathcal{J} \approx \mathcal{J}_0\{\sigma I(I+1)\}^{1/3} . \tag{4.53}$$

Substituting this value of $\mathcal{J}$ into (4.36) gives

$$E_{rot}^{(|\sigma| \to \infty)} \approx \frac{\{I(I+1)\}^{2/3}}{2\mathcal{J}_0\sigma^{1/3}} + \frac{B\sqrt{I(I+1)}}{2\mathcal{J}_0^3\sigma} + \frac{C\{I(I+1)\}^{2/3}}{2\mathcal{J}_0^4\sigma^{4/3}} . \tag{4.54}$$

But since $r \approx \{\sigma I(I+1)\}^{1/3}$ and $|r| \to \infty$ the first term will dominate because it has the lowest power of σ in its denominator.

Thus

$$E_{rot}(|\sigma| \to \infty) \approx \frac{\{I(I+1)\}^{2/3}}{2\mathcal{J}_0\sigma^{1/3}} . \tag{4.55}$$

Hence

$$R_4(|\sigma| \to \infty) = \left(\frac{20}{6}\right)^{2/3} = 2.23 \quad . \tag{4.56}$$

Thus the range of validity of our model is the same as that of the original VMI model[12], namely $2.23 \leq R_4 \leq 10/3$.

4.3 Calculations and Results

We have evaluated the energy levels, E_{rot}, (using equations (4.24B)and (4.25B)) for the same 83 nuclei, covering the rotation, transition, and vibration regions, as considered in Reference 12. The results are presented in Table I. The parameters $\mathcal{J}_o$, B, and C, for each nucleus considered, were determined by a least-squares fitting procedure (see Appendix C) involving the energies of all the known spin states. In Table I, the first row gives our calculated energies, expressed in keV, for levels up to five states beyond those experimentally known. The second row gives the experimental energies, which were taken from Reference 12, and the thied row gives the energies as calculated using the VMI model[12]. The R_4 value ($R_4 = E(I = 4+)/E(I = 2+)$) for each nucleus is also included.

As can be seen, the results are excellent for all regions (rotation, transition, and vibration), being, in the vast majority of cases, better than those predicted by the VMI model. This is due primarily to the improved fitting of the high spin states. The above conclusion is born out by a comparison of the weighted sum of squares (WSS) for our values with those given by the VMI model. (WSS = $\sum_I \left[E_{exp}(I) - E_{theor}(I)\right]^2 / E_{exp}(I)$). This is presented in Table II where the first column gives our WSS value for each nucleus and the second column gives the WSS values for the VMI model. For almost all cases our WSS values are at least an order of magnitude smaller than those obtained on the basis of the VMI model.

TABLE I COMPARISON OF ROTATIONAL ENERGY LEVELS

FOR EACH NUCLEUS THE FIRST ROW CONTAINS THE ENERGIES OBTAINED WITH OUR MODEL. THE SECOND ROW GIVES THE EXPERIMENTAL ENERGIES AND THE THIRD ROW GIVES ENERGIES PREDICTED BY THE VMI MODEL. ALL ENERGIES ARE IN KEV. (THE VALUES IN THE SECOND AND THIRD ROW ARE TAKEN FROM REFERENCE 18.) R(4)=E(I=4)/E(I=2)

SPIN	2	4	6	8	10	12	14	16	18	20	22	24	26
PD 108	(R(4) = 2.41)												
OUR	433.8	1047.5	1770.0	2575.0	3447.8	4378.8	5361.2	6389.9					
EXP	433.8	1047.5	1770.0										
VMI	434.0	1045.0	1772.8										
CD 110	(R(4) = 2.34)												
OUR	657.7	1542.2	2479.7	3461.6	4481.8	5535.7	6619.8	7731.7					
EXP	657.7	1542.3	2479.6										
VMI	661.4	1513.5	2510.0										
XE 120	(R(4) = 2.47)												
OUR	323.0	806.6	1406.3	2098.6	2869.5	3709.3	4610.8	5568.6	6578.4				
EXP	321.8	794.4	1396.0	2097.0									
VMI	319.4	807.0	1399.6	2072.0									
XE 122	(R(4) = 2.50)												
OUR	328.7	836.8	1473.3	2212.3	3038.3	3940.5	4911.2	5944.3	7035.0				
EXP	331.1	828.6	1467.0	2217.0									
VMI	328.4	842.6	1471.8	2188.3									
XE 124	(R(4) = 2.48)												
OUR	351.8	889.7	1560.9	2338.7	3206.7	4154.0	5172.4	6255.6	7398.5				
EXP	355.0	880.0	1555.0	2355.0									
VMI	351.8	897.2	1562.7	2319.3									
XE 126	(R(4) = 2.44)												
OUR	388.5	952.7	1646.1	2442.7	3326.6	4287.1	5316.3	6408.1	7557.7				
EXP	390.0	950.0	1645.0	2445.0									
VMI	388.0	959.9	1648.4	2426.1									
XE 128	(R(4) = 2.34)												
OUR	444.0	1041.0	1745.0	2531.8	3387.5	4302.6	5270.6	6286.3					
EXP	444.0	1041.0	1745.0										
VMI	443.8	1041.0	1745.4										
XE 130	(R(4) = 2.25)												
OUR	534.4	1200.0	1956.2	2782.4	3666.6	4601.1	5580.1	6599.3	7655.4				
EXP	534.0	1203.0	1951.0	2785.0									
VMI	534.2	1192.4	1955.7	2801.4									
BA 124	(R(4) = 2.83)												
OUR	226.8	652.6	1221.8	1910.3	2701.7	3564.3	4549.2	5589.7					

EXP	229.5	650.6	1223.0							
VMI	228.7	656.8	1215.2							
BA 126	(R(4) = 2.78)									
OUR	252.4	718.0	1342.0	2096.0	2961.8	3926.8	4981.3	6117.8	7330.2	
EXP	256.1	711.6	1333.0	2090.0						
VMI	253.4	725.1	1339.0	2060.0						
CE 128	(R(4) = 2.93)									
OUR	207.0	606.7	1155.8	1820.9	2586.3	3439.5	4371.2	5374.3	6443.0	
EXP	207.3	607.3	1157.8	1820.0						
VMI	206.2	613.1	1158.6	1809.7						
CE 130	(R(4) = 2.83)									
OUR	251.2	711.7	1322.6	2053.9	2886.9	3808.9	4810.4	5884.0	7024.0	
EXP	251.4	710.7	1324.1	2053.1						
VMI	252.4	719.5	1325.7	2036.8						
CE 132	(R(4) = 2.64)									
OUR	325.1	860.6	1540.0	2332.3	3219.3	4188.6	5231.2	6340.3	7510.3	
EXP	325.4	858.9	1542.7	2331.0						
VMI	324.1	865.9	1542.2	2320.0						
CE 134	(R(4) = 2.56)									
OUR	407.9	1051.2	1862.2	2807.4	3866.4	5025.3	6273.9	7604.3	9010.1	
EXP	409.2	1048.6	1862.0	2809.0						
VMI	407.0	1060.5	1866.1	2786.5						
CE 136	(R(4) = 2.38)									
OUR	552.0	1313.6	2213.0	3218.2	4310.8	5478.9	6713.7	8008.8		
EXP	552.0	1313.6	2213.0							
VMI	552.2	1313.0	2215.1							
SM 150	(R(4) = 2.35)									
OUR	330.1	774.7	1270.2	1804.6	2371.1	2965.3	3583.9	4224.7		
EXP	330.0	775.0	1270.0							
VMI	331.1	767.0	1279.0							
SM 152	(R(4) = 3.01)									
OUR	121.8	367.3	706.1	1127.1	1612.8	2156.8	2753.1	3396.9	4084.1	4811.7
EXP	121.8	366.4	712.0	1122.0	1615.0					
VMI	121.0	369.9	712.3	1127.3	1601.8					
SM 154	(R(4) = 3.26)									
OUR	81.4	266.1	548.7	924.7	1389.8	1940.4	2573.1	3285.0	4073.3	
EXP	81.9	267.0	545.0	927.0						
VMI	81.5	267.7	550.4	920.3						
GD 152	(R(4) = 2.20)									
OUR	340.1	766.3	1275.7	1846.9	2472.2	3144.9	3860.3	4614.5		
EXP	344.2	755.6	1285.0							
VMI	341.9	769.7	1267.2							

GD 154	(R(4) = 3.02)									
OUR	123.0	371.4	717.9	1145.9	1644.1	2204.3	2820.2	3487.0	4200.6	4957.8
EXP	123.1	371.2	718.1	1146.0	1644.0					
VMI	122.0	374.4	722.8	1146.0	1630.7					
GD 156	(R(4) = 3.24)									
OUR	88.9	288.1	584.6	966.0	1421.6	1942.9	2523.1	3156.5	3838.8	
EXP	88.9	288.2	584.5	966.0						
VMI	88.8	288.4	585.0	965.2						
GD 158	(R(4) = 3.29)									
OUR	79.4	261.9	538.6	898.4	1331.0	1827.6	2381.3	2986.2	3637.7	
EXP	79.5	261.5	539.0	898.2						
VMI	79.6	261.4	537.8	899.5						
GD 160	(R(4) = 3.28)									
OUR	75.2	246.6	510.2	862.2	1299.2	1818.2	2416.4	3091.2	3840.3	
EXP	75.3	247.0	509.0	863.0						
VMI	75.1	247.5	511.5	860.0						
DY 154	(R(4) = 2.23)									
OUR	336.4	746.5	1220.6	1744.9	2311.4	2914.4	3550.0	4215.2	4907.4	5624.7
EXP	334.7	747.0	1224.4	1748.2	2305.5					
VMI	333.7	744.8	1221.5	1749.7	2321.1					
DY 156	(R(4) = 2.92)									
OUR	137.9	403.6	767.1	1211.5	1726.0	2302.7	2935.7	3620.3	4352.7	
EXP	138.0	403.0	766.0	1212.0						
VMI	137.1	407.3	769.3	1201.3						
DY 158	(R(4) = 3.20)									
OUR	99.1	316.5	633.9	1036.5	1512.3	2052.4	2649.9	3299.2	3995.9	4736.3
EXP	99.0	317.0	633.0	1037.0	1512.0					
VMI	98.8	317.0	635.3	1036.9	1509.2					
DY 160	(R(4) = 3.28)									
OUR	86.7	283.9	582.3	971.6	1442.2	1985.6	2594.8	3264.0	3988.3	4763.4
EXP	86.7	284.0	582.0	972.0	1442.0					
VMI	86.7	284.0	582.6	971.7	1445.5					
DY 162	(R(4) = 3.28)									
OUR	81.1	265.7	546.3	922.9	1384.0	1926.5	2545.4	3236.4	3995.5	
EXP	81.0	266.0	546.0	923.0						
VMI	80.9	266.2	549.2	921.5						
DY 164	(R(4) = 3.30)									
OUR	73.4	242.2	500.1	839.0	1250.8	1728.1	2264.6	2855.0	3494.9	
EXP	73.4	242.2	500.1	839.0						
VMI	73.5	242.1	500.1	840.3						
ER 156	(R(4) = 2.32)									
OUR	344.5	798.4	1342.0	1957.4	2633.7	3363.3	4140.9	4962.0	5823.4	
EXP	344.4	797.3	1340.5	1958.7						
VMI	343.3	802.4	1343.2	1947.3						

ER 158	(R(4) = 2.74)										
OUR	191.0	533.5	974.3	1489.7	2066.1	2694.8	3369.5	4085.5	4839.2	5627.5	6448.1
EXP	192.7	526.4	972.3	1496.0	2075.7	2684.4					
VMI	191.3	535.1	975.8	1489.5	2062.7	2686.7					
ER 160	(R(4) = 3.09)										
OUR	125.2	392.7	768.3	1228.7	1758.6	2347.7	2988.6	3673.6	4404.4	5171.5	5974.0
EXP	126.2	390.5	766.8	1231.4	1763.5	2342.9					
VMI	125.8	392.5	767.1	1227.4	1758.4	2350.0					
ER 162	(R(4) = 3.24)										
OUR	100.9	327.2	662.2	1089.3	1595.4	2170.2	2805.8	3496.0	4235.8	5021.2	
EXP	101.0	327.0	662.0	1090.0	1595.0						
VMI	101.0	327.1	661.7	1089.1	1596.4						
ER 164	(R(4) = 3.27)										
OUR	91.4	297.1	607.9	1015.3	1511.3	2089.0	2742.5	3466.4	4256.2	5107.9	
EXP	91.0	298.0	608.0	1014.0	1512.0						
VMI	90.9	297.6	610.0	1016.7	1507.1						
ER 166	(R(4) = 3.29)										
OUR	80.6	265.0	544.9	910.1	1350.9	1859.1	2427.8	3051.3	3724.8		
EXP	80.6	264.9	545.0	910.0							
VMI	80.6	264.8	544.6	910.6							
ER 168	(R(4) = 3.31)										
OUR	79.8	264.0	548.7	928.4	1397.1	1948.7	2577.4	3277.9			
EXP	79.8	264.0	549.0								
VMI	79.8	264.1	548.9								
ER 170	(R(4) = 3.30)										
OUR	79.9	261.0	542.1	917.2	1381.2	1928.7	2554.6	3254.1			
EXP	79.0	261.0	542.0								
VMI	79.0	261.1	541.9								
YB 158	(R(4) = 2.33)										
OUR	357.9	833.8	1382.4	1986.6	2636.7	3326.2	4050.5	4806.4			
EXP	357.9	833.9	1382.2								
VMI	358.3	830.8	1385.9								
YB 160	(R(4) = 2.63)										
OUR	242.8	639.7	1145.0	1736.8	2401.7	3130.7	3917.1	4755.8	5642.5		
EXP	243.0	638.3	1147.1	1735.8							
VMI	241.8	644.9	1147.6	1725.4							
YB 162	(R(4) = 2.92)										
OUR	165.6	490.0	922.5	1437.1	2018.4	2656.7	3344.8	4077.5	4850.8	5661.1	
EXP	166.5	486.7	922.9	1444.1	2013.5						
VMI	166.0	490.2	922.3	1436.6	2017.8						
YB 164	(R(4) = 3.13)										
OUR	122.2	385.1	757.7	1217.4	1749.1	2342.3	2989.4	3684.7	4424.9	5202.2	

EXP	122.5	384.0	758.0	1219.0	1748.0							
VMI	122.4	384.9	757.3	1217.0	1750.0							
YB 166	(R(4) = 3.24)											
OUR	101.5	330.2	667.9	1096.5	1602.0	2173.6	2803.4	3485.2	4214.0	4985.9	5797.5	
EXP	101.8	329.7	667.1	1097.0	1604.0	2172.0						
VMI	101.8	329.9	667.4	1097.0	1602.0	2173.0						
YB 168	(R(4) = 3.26)											
OUR	86.8	284.5	581.9	966.5	1427.3	1955.2	2542.8	3184.2	3874.6	4610.0		
EXP	87.0	284.0	582.0	967.0	1427.0							
VMI	87.0	284.3	581.2	966.1	1428.7							
YB 170	(R(4) = 3.30)											
OUR	84.3	277.3	572.3	962.0	1438.9	1995.9	2626.8	3326.1	4088.6	4910.6		
EXP	84.2	277.7	572.0	962.0	1439.0							
VMI	84.2	277.4	572.9	962.4	1437.7							
YB 172	(R(4) = 3.31)											
OUR	78.4	260.8	540.7	908.2	1352.9	1866.0	2439.9	3068.4	3746.6	4470.2		
EXP	78.7	260.3	540.0	910.0	1352.0							
VMI	78.9	260.3	538.6	906.6	1357.0							
YB 174	(R(4) = 3.29)											
OUR	76.3	252.9	526.3	892.0	1344.7	1879.0	2469.5	3171.3	3920.0			
EXP	76.5	252.0	527.0	892.0								
VMI	76.3	252.9	526.3	891.9								
YB 176	(R(4) = 3.29)											
OUR	81.7	271.2	562.6	947.5	1416.8	1962.1	2575.9	3252.0	3985.2			
EXP	82.1	270.0	564.0	947.0								
VMI	82.0	271.0	561.9	948.2								
HF 166	(R(4) = 2.97)											
OUR	157.0	475.7	899.5	1400.0	1962.0	2575.5	3234.0	3932.3	4666.8	5434.2	6232.2	
EXP	158.7	470.7	897.6	1407.0	1971.0	2565.0						
VMI	158.2	474.1	897.1	1399.0	1963.0	2579.0						
HF 168	(R(4) = 3.11)											
OUR	123.1	386.9	757.1	1209.9	1730.2	2307.6	2935.5	3607.7	4320.2	5069.6	5853.2	
EXP	123.9	385.0	756.1	1212.0	1734.0	2304.0						
VMI	123.7	386.5	756.1	1209.0	1731.0	2309.0						
HF 170	(R(4) = 3.21)											
OUR	98.7	322.2	643.9	1041.3	1499.4	2008.5	2561.7	3153.0	3781.0	4440.1	5128.6	5844.4	6585.7
EXP	100.0	320.6	641.1	1041.0	1503.0	2013.0	2564.0	3147.0					
VMI	99.9	321.1	641.8	1041.0	1501.0	2011.0	2563.0	3151.0					
HF 172	(R(4) = 3.26)											
OUR	93.7	308.7	628.9	1036.2	1516.4	2058.9	2655.6	3300.5	3989.0	4717.1	5481.7	6280.3	
EXP	94.5	307.9	627.0	1036.0	1519.0	2063.0	2651.0						
VMI	94.4	308.2	627.6	1036.0	1518.0	2060.0	2655.0						

HF 174	(R(4) = 3.28)											
OUR	91.3	297.4	608.0	1012.4	1500.8	2064.9	2697.7	3393.2	4146.6	4953.5		
EXP	90.9	298.0	609.0	1010.0	1502.0							
VMI	91.0	297.5	608.8	1013.1	1499.5							
HF 176	(R(4) = 3.28)											
OUR	88.3	290.0	596.6	998.1	1485.1	2049.1	2682.9	3380.4	4136.3			
EXP	88.3	290.0	596.6	998.0								
VMI	88.1	289.6	596.2	998.3								
HF 178	(R(4) = 3.29)											
OUR	93.2	306.9	632.4	1059.0	1576.3	2174.8	2846.7	3585.2	4384.8			
EXP	93.2	306.8	632.5	1059.0								
VMI	93.2	306.7	632.1	1059.5								
HF 180	(R(4) = 3.31)											
OUR	93.3	308.6	641.1	1085.0	1633.7	2280.8	3019.8	3845.0	4751.0			
EXP	93.3	308.6	641.1	1084.9								
VMI	93.3	308.7	641.3	1084.7								
W 172	(R(4) = 3.07)											
OUR	122.0	379.3	728.2	1144.2	1613.6	2127.8	2680.8	3268.2	3886.7	4533.6	5206.7	5904.3
EXP	122.9	376.9	727.2	1147.0	1616.0	2129.0	2677.0					
VMI	123.8	376.1	720.9	1137.1	1611.8	2136.4	2704.8					
W 174	(R(4) = 3.17)											
OUR	111.5	356.0	704.8	1135.6	1633.3	2187.9	2792.0	3440.2	4126.1	4852.4	5610.4	
EXP	111.9	355.0	704.2	1137.0	1635.0	2186.0						
VMI	112.2	354.9	701.8	1132.9	1634.4	2196.4						
W 176	(R(4) = 3.21)											
OUR	107.9	350.2	701.0	1137.5	1644.1	2209.9	2827.1	3489.9	4193.9	4935.3	5711.4	
EXP	108.7	348.5	699.4	1140.0	1648.0	2206.0						
VMI	108.5	349.4	699.9	1138.0	1645.0	2210.0						
W 178	(R(4) = 3.29)											
OUR	102.8	343.4	700.7	1150.2	1674.2	2260.4	2900.2	3587.3	4316.7	5084.6	5887.8	6723.7
EXP	104.0	342.0	697.0	1152.0	1679.0	2264.0	2894.0					
VMI	105.4	341.4	690.7	1136.9	1666.5	2269.1	2936.5					
W 180	(R(4) = 3.29)											
OUR	100.9	338.0	692.6	1141.2	1666.2	2255.0	2899.0	3591.5	4327.5	5103.0	5914.8	
EXP	102.0	336.0	690.0	1147.0	1667.0	2252.0						
VMI	103.1	335.7	683.5	1131.5	1667.0	2279.8						
W 182	(R(4) = 3.29)											
OUR	100.0	329.8	679.9	1138.2	1692.8	2333.3	3050.9	3838.4	4689.8			
EXP	100.1	329.4	680.4	1138.0								
VMI	100.0	329.2	678.9	1138.0								
W 184	(R(4) = 3.27)											
OUR	111.1	364.1	747.2	1246.9	1850.5	2546.9	3327.2	4183.7				
EXP	111.2	364.0	748.2									
VMI	111.1	364.4	747.8									

W 186	(R(4) = 3.26)										
OUR	122.3	399.7	817.2	1358.6	2009.1	2756.7	3591.2	4504.6			
EXP	122.5	399.0	818.0								
VMI	122.4	399.8	817.2								
OS 178	(R(4) = 3.02)										
OUR	131.3	398.9	760.4	1192.5	1681.7	2219.2	2798.9	3416.1	4067.3	4749.6	5460.8
EXP	131.6	397.7	760.8	1193.7	1681.7	2218.5					
VMI	131.9	397.5	757.8	1190.8	1683.3	2226.5					
OS 180	(R(4) = 3.09)										
OUR	130.6	413.4	796.2	1250.4	1760.7	2317.7	2915.1	3548.2	4213.4	4908.0	5629.7
EXP	132.2	408.5	795.1	1257.3	1767.5	2308.5					
VMI	133.4	407.8	785.5	1234.2	1766.6	2346.1					
OS 182	(R(4) = 3.15)										
OUR	125.9	403.3	793.8	1269.9	1814.5	2416.5	3068.4	3764.5	4500.3	5272.3	
EXP	126.9	400.2	793.9	1276.9	1809.6						
VMI	127.3	400.5	788.7	1268.9	1825.8						
OS 184	(R(4) = 3.20)										
OUR	119.8	383.7	773.8	1274.7	1873.9	2561.2	3328.3	4168.6	5076.3		
EXP	119.8	383.6	773.9	1274.6							
VMI	119.4	385.0	775.4	1271.1							
OS 186	(R(4) = 3.16)										
OUR	137.1	434.3	868.2	1420.7	2077.5	2827.5	3661.8	4573.3	5556.0		
EXP	137.2	433.9	868.7	1420.5							
VMI	136.6	436.3	870.6	1415.8							
OS 188	(R(4) = 3.08)										
OUR	154.8	478.5	936.9	1514.0	2188.1	2949.8	3790.2	4702.1	5679.9		
EXP	155.0	477.9	939.8	1513.6							
VMI	154.3	481.4	941.5	1506.9							
OS 190	(R(4) = 2.93)										
OUR	186.3	549.0	1048.2	1661.5	2374.1	3175.3	4057.1	5012.8	6037.4		
EXP	186.7	547.8	1050.0	1662.0							
VMI	185.3	554.6	1052.4	1648.5							
PT 182	(R(4) = 2.71)										
OUR	154.1	426.3	784.1	1211.5	1698.3	2237.5	2824.1	3453.8	4123.5		
EXP	153.7	416.2	771.4	1202.4							
VMI	152.0	425.1	775.2	1183.3							
PT 184	(R(4) = 2.68)										
OUR	161.1	437.7	797.3	1223.9	1707.6	2241.7	2821.0	3441.7	4100.5	4795.0	
EXP	162.1	434.8	797.3	1228.9	1704.7						
VMI	160.2	443.1	803.4	1221.8	1687.6						
PT 186	(R(4) = 2.56)										
OUR	183.7	469.3	881.9	1344.3	1866.0	2440.0	3061.0	3724.7	4426.0	5168.0	

EXP	191.1	489.6	876.8	1341.1	1855.7						
VMI	188.4	500.2	888.1	1333.4	1825.7						
PT 188	(R(4) = 2.52)										
OUR	265.3	673.2	1183.3	1775.0	2435.8	3157.4	3933.4	4759.0			
EXP	265.9	671.3	1184.6								
VMI	265.2	676.4	1178.2								
PT 190	(R(4) = 2.51)										
OUR	291.3	732.4	1281.5	1916.7	2624.9	3397.1	4226.7	5108.8	6039.1	7014.3	
EXP	292.0	733.0	1283.0	1903.0	2636.0						
VMI	290.1	740.0	1288.9	1913.0	2598.9						
PT 192	(R(4) = 2.48)										
OUR	315.4	790.9	1381.8	2064.8	2826.0	3655.6	4546.7	5493.7	6492.4		
EXP	317.0	785.0	1388.0	2063.0							
VMI	315.0	797.2	1383.7	2049.3							
PT 194	(R(4) = 2.47)										
OUR	328.2	812.6	1410.7	2099.5	2865.2	3698.3	4591.9	5540.6	6540.1		
EXP	328.5	811.1	1411.6	2099.4							
VMI	327.1	818.6	1413.2	2086.5							
TH 228	(R(4) = 3.25)										
OUR	57.5	186.2	377.3	621.7	912.1	1243.0	1609.7	2008.7			
EXP	57.5	186.6	378.0								
VMI	57.5	186.5	378.1								
TH 232	(R(4) = 3.27)										
OUR	50.0	162.6	332.8	556.0	827.8	1144.4	1502.7	1899.6	2332.7	2799.8	
EXP	49.8	163.0	333.0	555.0	828.0						
VMI	49.7	162.9	333.9	556.6	825.2						
U 232	(R(4) = 3.29)										
OUR	47.7	156.4	321.4	537.2	798.3	1100.1	1438.8	1811.0			
EXP	47.6	156.6	321.0								
VMI	47.6	156.3	321.2								
U 234	(R(4) = 3.30)										
OUR	43.5	143.5	296.6	498.7	745.7	1033.6	1358.9	1718.6			
EXP	43.5	143.5	296.6								
VMI	43.5	143.5	296.6								
U 236	(R(4) = 3.28)										
OUR	45.2	149.4	311.4	529.8	803.6	1131.4	1512.3	1945.1			
EXP	45.3	148.7	312.0								
VMI	45.1	149.6	311.2								
U 238	(R(4) = 3.31)										
OUR	44.7	148.2	308.6	522.9	787.7	1099.6	1455.4	1851.9	2286.3	2756.3	3259.5
EXP	44.7	148.0	309.0	523.0	787.0	1100.0					
VMI	44.7	148.2	308.4	522.5	787.6	1100.3					

PU 238	(R(4) = 3.31)								
OUR	44.1	146.1	303.7	514.1	774.0	1080.1	1429.2	1818.4	2245.1
EXP	44.1	146.0	303.6	514.0					
VMI	44.1	146.0	303.6	514.0					
PU 240	(R(4) = 3.30)								
OUR	42.9	142.1	295.8	501.7	756.8	1058.1	1402.9	1788.4	
EXP	42.9	141.7	296.0						
VMI	42.8	142.0	295.7						
CM 242	(R(4) = 3.29)								
OUR	42.3	138.8	285.3	477.1	709.4	978.1	1279.8	1611.6	
EXP	42.2	139.0	285.0						
VMI	42.2	138.7	285.3						
CM 244	(R(4) = 3.32)								
OUR	42.9	142.3	296.2	502.3	757.5	1058.9	1403.6	1788.8	2212.1
EXP	42.9	142.3	296.0	502.0					
VMI	42.9	142.2	296.1	502.0					
CM 248	(R(4) = 3.31)								
OUR	43.4	143.9	299.9	509.3	769.3	1077.2	1430.2	1825.6	
EXP	43.4	143.6	300.0						
VMI	43.4	143.8	299.8						

TABLE II COMPARISON OF WSS VALUES

NUCLEUS	OUR WSS	VMI WSS	R(4)
PD 108	.222E-06	.105E-01	2.41
CD 110	.774E-05	.931E+00	2.34
XE 120	.114E+00	.525E+00	2.47
XE 122	.134E+00	.646E+00	2.50
XE 124	.244E+00	.944E+00	2.48
XE 126	.165E-01	.267E+00	2.44
XE 128	.955E-05	.182E-03	2.34
XE 130	.240E-01	.201E+00	2.25
BA 124	.945E-02	.112E+00	2.83
BA 126	.158E+00	.742E+00	2.78
CE 128	.737E-02	.120E+00	2.93
CE 130	.358E-02	.244E+00	2.83
CE 132	.929E-02	.114E+00	2.64
CE 134	.118E-01	.336E+00	2.56
CE 136	.501E-07	.234E-02	2.38
SM 150	.145E-03	.150F+00	2.35
SM 152	.497E-01	.172E+00	3.01
SM 154	.382E-01	.106E+00	3.26
GD 152	.328E+00	.525E+00	2.20
GD 154	.189E-03	.176E+00	3.02
GD 156	.508E-03	.134E-02	3.24
GD 158	.105E-02	.472E-02	3.29
GD 160	.447E-02	.243E-01	3.28
DY 154	.425E-01	.123E+00	2.23
DY 156	.278E-02	.160E+00	2.92
DY 158	.290E-02	.140E-01	3.20
DY 160	.374E-03	.921E-02	3.28
DY 162	.473E-03	.534E-02	3.28
DY 164	.193E-07	.219E-02	3.30
ER 156	.411E-02	.108E+00	2.32
ER 158	.181E+00	.219E+00	2.74
ER 160	.516E-01	.609E-01	3.09
ER 162	.848E-03	.214E-02	3.24
ER 164	.645E-02	.303E-01	3.27
ER 166	.826E-04	.727E-03	3.29
ER 168	.206E-03	.561E-04	3.31
ER 170	.220E-04	.568E-04	3.30
YB 158	.606E-04	.219E-01	2.33
YB 160	.754E-02	.137E+00	2.63
YB 162	.741E-01	.752E-01	2.92
YB 164	.673E-02	.841E-02	3.13
YB 166	.656E-02	.321E-02	3.24
YB 168	.151E-02	.428E-02	3.26
YB 170	.108E-02	.308E-02	3.30
YB 172	.738E-02	.353E-01	3.31
YB 174	.442E-02	.468E-02	3.29
YB 176	.105E-01	.132E-01	3.29
HF 166	.194E+00	.181E+00	2.97
HF 168	.340E-01	.296E-01	3.11
HF 170	.717E-01	.118E-01	3.21
HF 172	.348E-01	.120E-01	3.26
HF 174	.109E-01	.147E-01	3.28
HF 176	.178E-04	.136E-02	3.28
HF 178	.401E-04	.522E-03	3.29
HF 180	.134E-04	.132E-03	3.31
W 172	.398E-01	.474E+00	3.07
W 174	.104E-01	.735E-01	3.17

W 176	.392E-01	.193E-01	3.21
W 178	.757E-01	.100E+01	3.29
W 180	.680E-01	.626E+00	3.29
W 182	.100E-02	.353E-02	3.29
W 184	.141E-02	.743E-03	3.27
W 186	.231E-02	.247E-02	3.26
OS 178	.584E-02	.500E-01	3.02
OS 180	.179E+00	.117E+01	3.09
OS 182	.837E-01	.231E+00	3.15
OS 184	.460E-04	.190E-01	3.20
OS 186	.654E-03	.356E-01	3.16
OS 188	.194E-02	.615E-01	3.08
OS 190	.577E-02	.210E+00	2.93
PT 182	.125E+00	.531E+00	2.71
PT 184	.511E-01	.440E+00	2.68
PT 186	.211E+00	.942E+00	2.56
PT 188	.843E-02	.752E-01	2.52
PT 190	.150E+00	.681E+00	2.51
PT 192	.819E-01	.307E+00	2.48
PT 194	.365E-02	.156E+00	2.47
TH 228	.214E-02	.800E-04	3.25
TH 232	.368E-02	.168E-01	3.27
U 232	.891E-03	.699E-03	3.29
U 234	.145E-04	0.	3.30
U 236	.490E-02	.838E-02	3.28
U 238	.167E-02	.245E-02	3.31
PU 238	.103E-03	0.	3.31
PU 240	.114E-02	.117E-02	3.30
CM 242	.856E-03	.963E-03	3.29
CM 244	.339E-03	.104E-03	3.32
CM 248	.645E-03	.412E-03	3.31

In many cases the results of the VMI model lie outside the limits of experimental error while ours lie within. (A listing of the experimental errors for the nuclei considered is presented in Reference 12.) Thus if we take, as the experimental energy values for a given set of levels, the outside experimental limit to calculate the WSS (still using the same parameters), the WSS value obtained is not appreciably different, whereas an appreciable difference would be found for the VMI model for those cases where the calculated energies lie outside the limits of experimental error.

It may be argued that the improved fitting of the energy levels obtained in our case is due to the appearance of an extra parameter compared to that of the VMI model. However, the addition of any arbitrary term to the VMI model would bring only an insignificant improvement. The fact that our choice of the new additive term, depending on the parameter B, does improve the WSS values, in many cases by four or five orders of magnitude, suggests that our special choice of this term extends the VMI model in the most consistent manner.

It should be stressed at this point that our model is quite different from the extended cranking model due to Harris. It does not give the same results as would be obtained simply by taking the next two terms of the Harris series as an additional correction. This is evident from the great difference between our values of $\mathcal{J}_0$ and C as compared to the values of those parameters which enter formally in the same way in the Harris model.

The values of the parameters γ_0, B, and C obtained for the various cases are presented in Columns 3-5 of Table III. It is seen that generally $\gamma_0 \sim 10^{-2}$ keV^{-1}, B $\sim \pm 10^{-5}$ keV^{-2}, and C $\sim 10^{-7}$ keV^{-3}. For most nuclei, the values of γ_0 and C are comparable to those predicted by the VMI model. Two notable exceptions are Gd^{152} ($R_4 = 2.20$) and Er^{156} ($R_4 = 2.32$) for which our theory gives $\gamma_0 < 0$. This is in accord with the discussion in Section 4.2 where it was pointed out that for nuclei with $R_4 \sim 2.2$ we have $\sigma^- \to -\infty$ which, from (4.52), implies $r < 0$. But since $\gamma > 0$, this means that for this case, $\gamma_0 < 0$. With respect to the results for the new parameter B, there appears to be no preference in the sign (+ or -), although, as previously mentioned, this is of little consequence since B appears as B^2 in most places.

Column 6 of Table III gives the values of the softness parameter, σ^-, as predicted by our model. With a few exceptions, most of the nuclei in the rotation region ($3.0 < R_4 \leq 3.33$; far from closed-shell nuclei) have approximately the same value for as predicted by the VMI model, since, for these nuclei, B is quite small ($\sim 10^{-6}$ keV^{-2}) and the correction to the nuclear softness is negligible. For most nuclei in the transition and vibration regions (nuclei near and approaching closed shells) our model predicts $\sigma^- < 0$. This is seen more clearly from figure 3 in which we have plotted σ^- (on a logarithmic scale) versus the mass number A. The numbers in brackets refer to the R_4 values of the given nuclei. We note that in the region A = 92 to A = 112, $\sigma^- \approx \sigma_{VMI}$, and that the curves show the discontinuities at N = 98, 104, and 108 as discussed by Mariscotti et al.[12]

TABLE III LIST OF PARAMETERS EMPLOYED FOR THREE-PARAMETER FIT AND THE SOFTNESS SIGMA DERIVED FROM THEM

IS IN UNITS OF 1/KEV, B IN 1/(KEV**2), AND C IN 1/(KEV**3).
SIGMA=(2β_0C - 9(B**2))/(β_0 **4). R(4)=E(I=4)/E(I=2)

NUCLEUS	R(4)	β_0	B	C	SIGMA
PD 108	2.41	.483E-02	-.633E-05	.663E-07	.514E+00
CD 110	2.34	.389E-01	-.867E-04	.216E-06	-.222E-01
XE 120	2.47	.213E-02	.130E-04	.251E-07	-.694E+02
XE 122	2.50	.292E-02	.112E-04	.183E-07	-.139E+02
XE 124	2.48	.246E-02	.102E-04	.164E-07	-.232E+02
XE 126	2.44	.105E-02	.100E-04	.183E-07	-.709E+03
XE 128	2.34	.198E-02	-.603E-07	.555E-07	.142E+02
XE 130	2.25	.512E-02	-.151E-04	.931E-07	-.159E+01
BA 124	2.83	.936E-02	.994E-05	.911E-08	-.937E-01
BA 126	2.78	.839E-02	.837E-05	.715E-08	-.103E+00
CE 128	2.93	.118E-01	.698E-05	.171E-07	-.184E-02
CE 130	2.83	.868E-02	.704E-05	.154E-07	-.317E-01
CE 132	2.64	.501E-02	.654E-05	.196E-07	-.299E+00
CE 134	2.56	.274E-02	.672E-05	.809E-08	-.638E+01
CE 136	2.38	.247E-02	-.135E-05	.287E-07	.339E+01
SM 150	2.35	.230E-01	-.938E-04	.563E-06	-.191E+00
SM 152	3.01	.213E-01	.140E-04	.641E-07	.466E-02
SM 154	3.26	.360E-01	.676E-05	.140E-08	-.185E-03
GD 152	2.20	-.353E-02	.204E-04	.760E-07	-.276E+02
GD 154	3.02	.210E-01	.154E-04	.454E-07	-.126E-02
GD 156	3.24	.331E-01	.212E-05	.690E-07	.378E-02
GD 158	3.29	.380E-01	-.473E-05	.958E-07	.340E-02
GD 160	3.28	.391E-01	.647E-05	.119E-08	-.121E-03
DY 154	2.23	.206E-04	-.185E-05	.219E-06	-.120E+09
DY 156	2.92	.170E-01	.206E-04	.304E-07	-.334E-01
DY 158	3.20	.292E-01	.407E-05	.693E-07	.538E-02
DY 160	3.28	.343E-01	.681E-06	.508E-07	.253E-02
DY 162	3.28	.363E-01	.463E-05	.156E-07	.542E-03
DY 164	3.30	.408E-01	-.212E-05	.776E-07	.226E-02
ER 156	2.32	-.178E-02	.172E-04	.544E-07	-.285E+03
ER 158	2.74	.126E-01	.276E-05	.108E-06	.105E+00
ER 160	3.09	.235E-01	-.278E-05	.104E-06	.159E-01
ER 162	3.24	.295E-01	-.120E-05	.701E-07	.546E-02
ER 164	3.27	.319E-01	.536E-05	.194E-07	.941E-03
ER 166	3.29	.372E-01	-.190E-05	.737E-07	.285E-02
ER 168	3.31	.375E-01	0.	.275E-07	.104E-02
ER 170	3.30	.377E-01	.140E-05	.210E-07	.775E-03
YB 158	2.33	.711E-02	-.214E-04	.204E-06	-.485E+00
YB 160	2.63	.592E-02	.152E-04	.339E-07	-.137E+01
YB 162	2.92	.166E-01	-.630E-06	.983E-07	.428E-01
YB 164	3.13	.240E-01	-.173E-05	.927E-07	.133E-01
YB 166	3.24	.296E-01	-.390E-05	.853E-07	.638E-02
YB 168	3.26	.345E-01	-.213E-05	.773E-07	.375E-02
YB 170	3.30	.352E-01	.164E-05	.301E-07	.137E-02
YB 172	3.31	.388E-01	-.709E-05	.869E-07	.277E-02
YB 174	3.29	.392E-01	0.	.269E-07	.893E-03
YB 176	3.29	.369E-01	-.353E-05	.546E-07	.211E-02
HF 166	2.97	.195E-01	-.110E-04	.149E-06	.326E-01
HF 168	3.11	.241E-01	-.449E-05	.117E-06	.162E-01
HF 170	3.21	.318E-01	-.171E-04	.204E-06	.101E-01
HF 172	3.26	.327E-01	-.922E-05	.117E-06	.597E-02
HF 174	3.28	.323E-01	.290E-05	.400E-07	.233E-02

HF 176	3.28	.337E-01	.208E-06	.432E-07	.225E-02
HF 178	3.29	.321E-01	-.102E-05	.402E-07	.242E-02
HF 180	3.31	.320E-01	.604E-06	.142E-07	.866E-03
W 172	3.07	.257E-01	-.178E-04	.248E-06	.227E-01
W 174	3.17	.271E-01	-.665E-05	.130E-06	.123E-01
W 176	3.21	.285E-01	-.939E-05	.126E-06	.966E-02
W 178	3.29	.309E-01	-.158E-04	.129E-06	.626E-02
W 180	3.29	.315E-01	-.154E-04	.122E-06	.568E-02
W 182	3.29	.300E-01	-.172E-05	.366E-07	.267E-02
W 184	3.27	.268E-01	0.	.256E-07	.266E-02
W 186	3.26	.243E-01	0.	.231E-07	.322E-02
OS 178	3.02	.223E-01	-.664E-05	.179E-06	.307E-01
OS 180	3.09	.253E-01	-.234E-04	.229E-06	.162E-01
OS 182	3.15	.248E-01	-.112E-04	.130E-06	.141E-01
OS 184	3.20	.240E-01	.444E-05	.207E-07	.245E-02
OS 186	3.16	.206E-01	.492E-05	.161E-07	.247E-02
OS 188	3.08	.176E-01	.621E-05	.191E-07	.342E-02
OS 190	2.93	.128E-01	.109E-04	.887E-08	-.319E-01
PT 182	2.71	.122E-01	.284E-04	.515E-07	-.271E+00
PT 184	2.68	.106E-01	.298E-04	.600E-07	-.530E+00
PT 186	2.56	.822E-02	.264E-04	.555E-07	-.117E+01
PT 188	2.52	.345E-02	.175E-04	.364E-07	-.176E+02
PT 190	2.51	.271E-02	.154E-04	.311E-07	-.363E+02
PT 192	2.48	.237E-02	.133E-04	.255E-07	-.466E+02
PT 194	2.47	.170E-02	.133E-04	.268E-07	-.179E+03
TH 228	3.25	.515E-01	-.100E-07	.329E-06	.482E-02
TH 232	3.27	.584E-01	.176E-04	.117E-06	.937E-03
U 232	3.29	.625E-01	-.100E-07	.300E-06	.246E-02
U 234	3.30	.686E-01	0.	.272E-06	.168E-02
U 236	3.28	.658E-01	.840E-05	.381E-08	-.709E-05
U 238	3.31	.671E-01	-.245E-05	.159E-06	.105E-02
PU 238	3.31	.670E-01	-.100E-07	.156E-06	.100E-02
PU 240	3.30	.698E-01	.100E-06	.142E-06	.835E-03
CM 242	3.29	.705E-01	0.	.417E-06	.238E-02
CM 244	3.32	.697E-01	-.100E-07	.145E-06	.856E-03
CM 248	3.31	.690E-01	-.100E-07	.120E-06	.731E-03

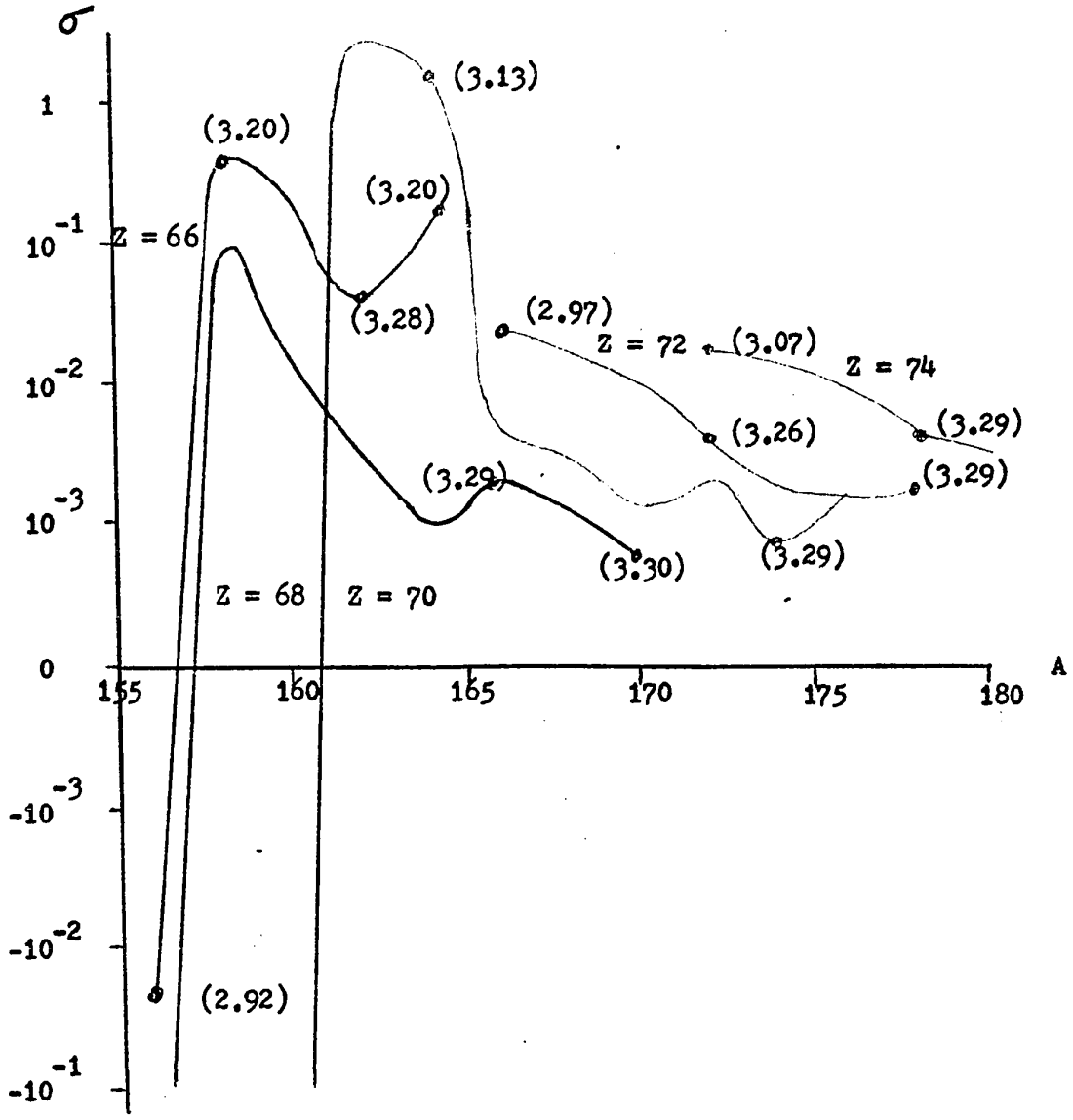

Figure 3. The nuclear softness, σ (on a logarithmic scale), plotted against mass number (A) for values of constant atomic number (Z). The numbers in brackets refer to R_4 values.

The anomalous behavior at N = 98 had been previously observed by Stephens et al.[1] who speculated, as a possible explanation for this effect, that the pairing correlations are reduced because of the large energy gap in the Nilsson diagram between the levels of $\frac{5}{2}^-$ (98 neutrons) and $\frac{7}{2}^+$. More recently, Duckworth[42] showed that breaks are also seen at N = 98 and N = 108 in the plot of double neutron separation energies as a function of neutron number. However, outside this region where $R_4 < 3$ and where N and/or Z begin to approach magic number values, the value of σ becomes negative. This transition in the value of σ occurs most noticeably in the region N = 88 to 90 where the nuclei go from an almost spherical shape to a well-deformed one.

The significance of "negative softness", which occurs in our model, as well as in the VMI model, is not yet understood. From the graphs presented in Figure 3 one can possibly conclude that the "degree of softness" is related to nuclear shell structure and thus to pairing forces in the nucleus. It would appear that a microscopic calculation might provide a better physical picture.

In Figure 4 we have plotted graphs of the effective moment of inertia, $\mathcal{J}$, versus nuclear spin, I, for a few representative nuclei from the rotation, transition, and vibration regions. The numbers in brackets give the R_4 values in each case. From these graphs it is seen that:

(i) the $\mathcal{J}_0$ values increase in a fairly regular fashion with increasing R_4,

(ii) for well-deformed, stable nuclei, e.g., Hf^{180}, $\mathcal{J}$ is almost constant as I increases,

(iii) the nuclei with the most dramatic relative increase of $\mathcal{J}$ with respect to I are those with small $\mathcal{J}_0$ and $R_4 \approx 2.23$,

(iv) despite the fact that many nuclei have a negative value of σ their $\mathcal{J}$ versus I curves resemble those having positive values of σ in as much as the slope is concerned,

(v) those nuclei which appear to have $d\mathcal{J}/dI < 0$ from the graph, have $\sigma > 0$.

A situation similar to that in (iv) above has been reported by Schraff-Goldhaber et al.[35] in which they predict $\sigma_{VMI} < 0$ for Te^{120} while their $\mathcal{J}$ versus I graph for this isotope resembles that of Xe^{120} for which $\sigma_{VMI} > 0$. It is hoped that once the concept of negative softness is understood that these paradoxes appearing both in our approach and in the VMI model may be resolved. A complete listing of the effective moments of inertia for each spin state for all the cases considered is given in the first column of Table IV. The values are given in order of increasing spin, I = 2+, 4+, ... (The value for spin I = 0 is not given as this is equal to $\mathcal{J}_0$, the values of which were presented in Table III.)

A measure of the discrepancy between $\mathcal{J}_0$ and $\mathcal{J}$ is δ, defined by $\delta = \frac{\mathcal{J} - \mathcal{J}_0}{\sigma}$. Values of δ for each spin state of each nucleus are presented in Column 2 of Table IV. As a general rule it is seen that high values of δ correspond to small R_4 values and vice versa.

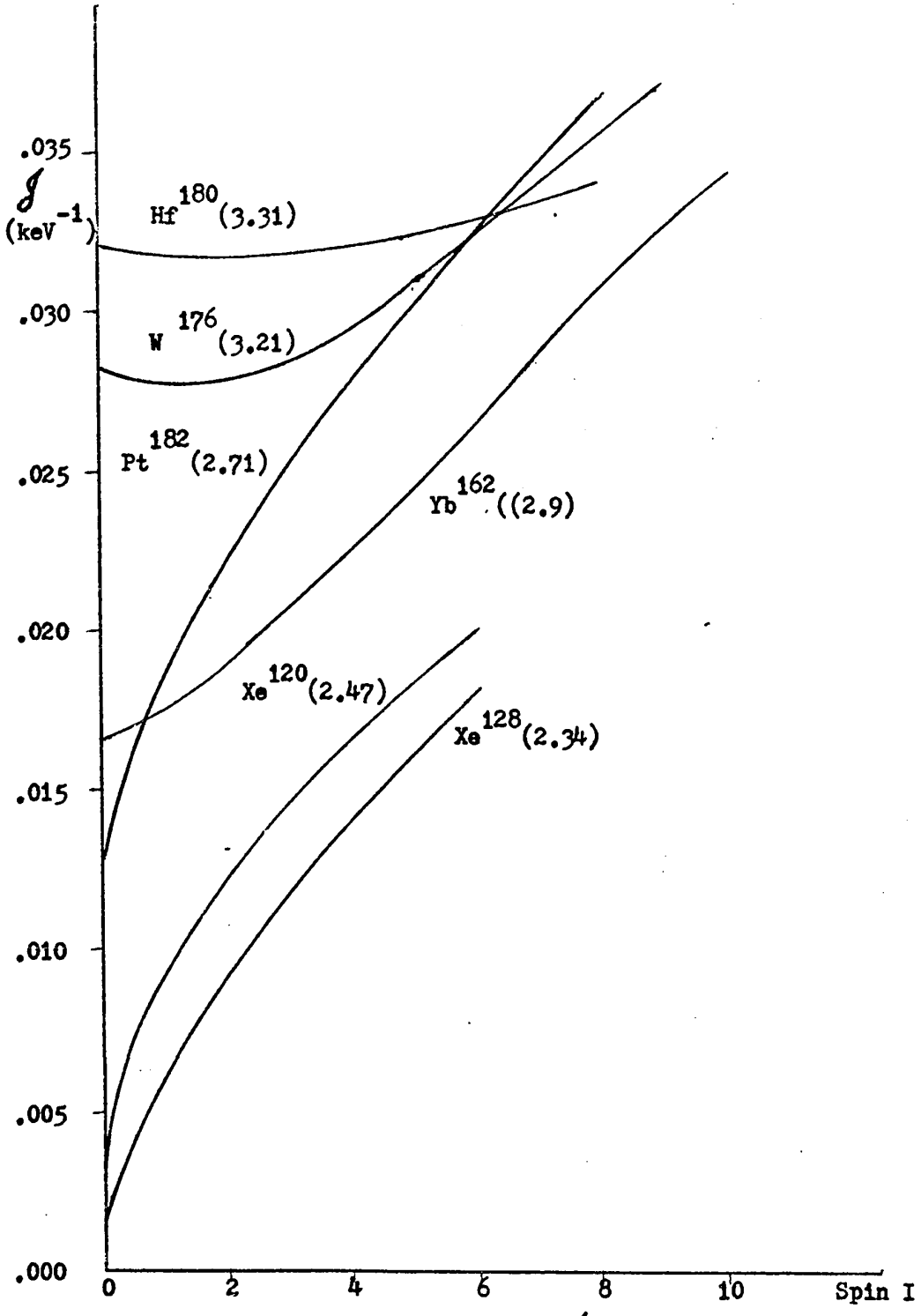

Figure 4. The effective moment of inertia, $\mathcal{J}$, plotted against nuclear spin I for representative nuclei from the rotation, transition, and vibration regions. The numbers in brackets refer to R_4 values.

TABLE IV LIST OF EFFECTIVE MOMENTS OF INERTIA AND DELTA. (DELTA = $(\mathcal{J} - \mathcal{J}_0)/\mathcal{J}$)

		$\mathcal{J}$	δ
PD108	(R(4) = 2.41)	.009190	.474290
		.013351	.638109
		.016953	.715000
CD110	(R(4) = 2.34)	.005844	-5.655146
		.009863	-2.943370
		.013529	-1.674902
XE120	(R(4) = 2.47)	.012102	.824149
		.016461	.870712
		.020034	.893773
		.023177	.908176
XE122	(R(4) = 2.50)	.011619	.748484
		.015572	.812337
		.018815	.844687
		.021668	.865134
XE124	(R(4) = 2.48)	.010943	.774931
		.014744	.832959
		.017862	.862116
		.020604	.880466
XE126	(R(4) = 2.44)	.010292	.897791
		.014184	.925840
		.017375	.939460
		.020182	.947879
XE128	(R(4) = 2.34)	.009430	.789655
		.013722	.855443
		.017372	.885820
XE130	(R(4) = 2.25)	.008208	.376709
		.012580	.593295
		.016380	.687657
		.019841	.742141
BA124	(R(4) = 2.83)	.014793	.367481
		.017928	.478075
		.020563	.544959

BA126	(R(4) = 2.78)		
		.013444	.375546
		.016338	.486169
		.018768	.552708
		.020926	.598822
CE128	(R(4) = 2.93)		
		.015852	.255735
		.018743	.370550
		.021323	.446716
		.023688	.501964
CE130	(R(4) = 2.83)		
		.013516	.357981
		.016600	.477257
		.019264	.549537
		.021666	.599480
CE132	(R(4) = 2.64)		
		.011186	.552324
		.014652	.658232
		.017577	.715102
		.020188	.751948
CE134	(R(4) = 2.56)		
		.009230	.702788
		.012253	.776113
		.014735	.813828
		.016919	.837856
CE136	(R(4) = 2.38)		
		.007411	.667132
		.010743	.770393
		.013597	.818579
SM150	(R(4) = 2.35)		
		.012120	-.697755
		.019036	-.208226
		.025176	.086438
SM152	(R(4) = 3.01)		
		.026361	.190185
		.030333	.296215
		.034012	.372353
		.037452	.430004
		.040696	.475436
SM154	(R(4) = 3.26)		
		.037318	.035970
		.038376	.062536
		.039386	.086593
		.040361	.108637
GD152	(R(4) = 2.20)		
		.013140	1.266337

		.019102	1.184581
		.024018	1.146800
GD154	(R(4) = 3.02)		
		.026107	.197058
		.029915	.299262
		.033377	.371950
		.036586	.427035
		.039597	.470604
GD156	(R(4) = 3.24)		
		.034233	.033948
		.035992	.081166
		.038137	.132841
		.040470	.182844
GD158	(R(4) = 3.29)		
		.037879	-.003074
		.038897	.023180
		.040612	.064412
		.042734	.110883
GD160	(R(4) = 3.28)		
		.040304	.029480
		.041248	.051687
		.042155	.072106
		.043035	.091066
DY154	(R(4) = 2.23)		
		.013480	.998469
		.020222	.998980
		.025954	.999205
		.031108	.999337
		.035866	.999425
DY156	(R(4) = 2.92)		
		.023958	.290342
		.028293	.399058
		.032010	.468839
		.035345	.518968
DY158	(R(4) = 3.20)		
		.030997	.059027
		.033304	.124224
		.035869	.187300
		.038562	.243627
		.041235	.292668
DY160	(R(4) = 3.28)		
		.034897	.018454
		.036068	.050334
		.037620	.089512
		.039405	.130743
		.041319	.171010

DY162	(R(4) = 3.28)	.037369	.027962
		.038368	.053273
		.039450	.079237
		.040594	.105186
DY164	(R(4) = 3.30)	.041021	.004214
		.041934	.025887
		.043363	.057981
		.045136	.094983
ER156	(R(4) = 2.32)	.012508	1.142317
		.017906	1.099411
		.022347	1.079654
		.026263	1.067778
ER158	(R(4) = 2.74)	.017838	.292310
		.022671	.443172
		.027034	.533044
		.031042	.593336
		.034781	.637050
		.038308	.670465
ER160	(R(4) = 3.09)	.024681	.049416
		.027576	.149203
		.030871	.240020
		.034192	.313846
		.037438	.373329
		.040583	.421891
ER162	(R(4) = 3.24)	.030096	.021096
		.031736	.071640
		.033895	.130812
		.036283	.188012
		.038751	.239734
ER164	(R(4) = 3.27)	.033316	.041756
		.034645	.078513
		.036063	.114763
		.037536	.149502
		.039037	.182203
ER166	(R(4) = 3.29)	.037425	.006924
		.038494	.034509
		.040097	.073090
		.042025	.115632
ER168	(R(4) = 3.31)	.037732	.006143

		.038252	.019653
		.039017	.038890
ER170	(R(4) = 3.30)		
		.038143	.011612
		.038744	.026955
		.039518	.046012
YB158	(R(4) = 2.33)		
		.011637	.389440
		.017437	.592513
		.022473	.683830
YB160	(R(4) = 2.63)		
		.015105	.607872
		.019740	.699934
		.023582	.748821
		.026978	.780445
YB162	(R(4) = 2.92)		
		.019488	.147197
		.023424	.290516
		.027272	.390611
		.030913	.462387
		.034360	.516319
YB164	(R(4) = 3.13)		
		.025242	.049188
		.027923	.140485
		.031010	.226052
		.034151	.297246
		.037239	.355514
YB166	(R(4) = 3.24)		
		.029817	.006387
		.031418	.057029
		.033690	.120607
		.036241	.182507
		.038884	.238079
		.041536	.286729
YB168	(R(4) = 3.26)		
		.034785	.009093
		.036055	.043982
		.037898	.090467
		.040054	.139440
		.042364	.186360
YB170	(R(4) = 3.30)		
		.035823	.017497
		.036706	.041119
		.037829	.069602
		.039125	.100413
		.040536	.131724

YB172	(R(4) = 3.31)		
		.038170	-.016989
		.038684	-.003470
		.039945	.028220
		.041693	.068954
		.043723	.112184
YB174	(R(4) = 3.29)		
		.039408	.005275
		.039877	.016969
		.040573	.033832
		.041454	.054377
YB176	(R(4) = 3.29)		
		.036710	-.005985
		.037234	.008190
		.038270	.035034
		.039665	.068968
HF166	(R(4) = 2.97)		
		.019962	.021765
		.023833	.180679
		.027931	.300868
		.031881	.387500
		.035649	.452244
		.039250	.502490
HF168	(R(4) = 3.11)		
		.025036	.037261
		.027961	.137971
		.031357	.231324
		.034793	.307242
		.038155	.368269
		.041413	.417968
HF170	(R(4) = 3.21)		
		.030331	-.049119
		.032478	.020235
		.035856	.112546
		.039561	.195656
		.043300	.265124
		.046982	.322711
		.050577	.370851
		.054079	.411593
HF172	(R(4) = 3.26)		
		.031993	-.023509
		.033241	.014929
		.035470	.076835
		.038130	.141231
		.040948	.200338
		.043805	.252478
		.046645	.297998
HF174	(R(4) = 3.28)		
		.033268	.029620
		.034586	.066668

		.036189	.107954
		.037955	.149453
		.039808	.189050
HF176	(R(4) = 3.28)		
		.034207	.014250
		.035193	.041876
		.036546	.077337
		.038134	.115767
HF178	(R(4) = 3.29)		
		.032349	.007112
		.033170	.031687
		.034400	.066307
		.035895	.105183
HF180	(R(4) = 3.31)		
		.032276	.009332
		.032752	.023731
		.033396	.042568
		.034176	.064412
W 172	(R(4) = 3.07)		
		.025190	-.019936
		.029165	.119068
		.033735	.238408
		.038250	.328299
		.042601	.396905
		.046781	.450797
		.050804	.494285
W 174	(R(4) = 3.17)		
		.027354	.010697
		.029883	.094411
		.033097	.182341
		.036461	.257792
		.039805	.320147
		.043075	.371761
W 176	(R(4) = 3.21)		
		.027955	-.019271
		.029902	.047126
		.032758	.130200
		.035886	.206013
		.039056	.270454
		.042187	.324600
W 178	(R(4) = 3.29)		
		.028776	-.075372
		.029660	-.043333
		.031945	.031300
		.034752	.109551
		.037717	.179532
		.040700	.239681
		.043651	.291075

W 180	(R(4) = 3.29)		
		.029298	-.073463
		.029998	-.048417
		.032092	.019988
		.034748	.094893
		.037592	.163355
		.040474	.222945
W 182	(R(4) = 3.29)		
		.030106	.002146
		.030833	.025659
		.031999	.061152
		.033444	.101716
W 184	(R(4) = 3.27)		
		.027215	.015241
		.028097	.046165
		.029304	.085454
W 186	(R(4) = 3.26)		
		.024752	.018278
		.025699	.054440
		.026968	.098933
OS178	(R(4) = 3.02)		
		.023985	.070888
		.026154	.208482
		.032528	.314906
		.036762	.393804
		.040811	.453953
		.044686	.501308
OS180	(R(4) = 3.09)		
		.023059	-.099237
		.026525	.044388
		.030822	.177615
		.035111	.278071
		.039257	.354307
		.043244	.413851
OS182	(R(4) = 3.15)		
		.024031	-.030144
		.026491	.065502
		.029763	.168236
		.033169	.253662
		.036532	.322359
OS184	(R(4) = 3.20)		
		.025645	.064361
		.027289	.120732
		.029030	.173460
		.030802	.221021
OS186	(R(4) = 3.16)		
		.022581	.087676
		.024390	.155314

		.026216	.214155
		.028020	.264763
OS188	(R(4) = 3.08)		
		.020349	.137378
		.022704	.226874
		.024964	.296849
		.027121	.352780
OS190	(R(4) = 2.93)		
		.017637	.276939
		.020662	.382809
		.023248	.451458
		.025563	.501134
PT182	(R(4) = 2.71)		
		.022628	.460765
		.028259	.568216
		.032942	.629601
		.037082	.670951
PT184	(R(4) = 2.68)		
		.022047	.518274
		.027987	.620522
		.032904	.677230
		.037242	.714826
		.041190	.742158
PT186	(R(4) = 2.56)		
		.019747	.583671
		.025508	.677710
		.030261	.728329
		.034449	.761356
		.038259	.785117
PT188	(R(4) = 2.52)		
		.014451	.760980
		.019417	.822110
		.023491	.852958
PT190	(R(4) = 2.51)		
		.013307	.796377
		.018004	.849503
		.021856	.876026
		.025244	.892662
		.028323	.904331
PT192	(R(4) = 2.48)		
		.012331	.807504
		.016719	.858030
		.020318	.883172
		.023442	.898916
PT194	(R(4) = 2.47)		
		.012037	.898697

		.016476	.896769
		.020116	.915445
		.023316	.927053
TH228	(R(4) = 3.25)		
		.052909	.026622
		.055733	.075952
		.059342	.132151
TH232	(R(4) = 3.27)		
		.060880	.041134
		.063278	.077477
		.065843	.113414
		.068510	.147925
		.071230	.180464
U 232	(R(4) = 3.29)		
		.063395	.014112
		.065311	.043043
		.067954	.080264
U 234	(R(4) = 3.30)		
		.069279	.009805
		.070770	.030662
		.072695	.058922
U 236	(R(4) = 3.28)		
		.066743	.014008
		.067511	.025218
		.068269	.036042
U 238	(R(4) = 3.31)		
		.067249	.002308
		.067990	.013178
		.069202	.030463
		.070794	.052263
		.072675	.076787
		.074765	.102605
PU238	(R(4) = 3.31)		
		.068202	.005896
		.069107	.018915
		.070443	.037521
		.072123	.059943
PU240	(R(4) = 3.30)		
		.070157	.005084
		.070947	.016172
		.072120	.032172
CM242	(R(4) = 3.29)		
		.071479	.013692
		.073579	.041843
		.076484	.078234

CM244 (R(4) = 3.32)

.070054	.005046
.070853	.016279
.072044	.032535
.073556	.052419

CM248 (R(4) = 3.31)

.069299	.004312
.069978	.013987
.070997	.028128

We have presented in Appendix B a list of the successive terms of (4.24B) for each spin state for all the nuclei considered. In the rotation region, the first term is dominant and the second two are either successively smaller or are in competition. (One exception is Er^{162} ($R_4 = 3.2$) for which the second term is the dominant term of the three.) This suggests that, at least for this region, the last two terms in our expression may represent corrections to the rigid rotor formula, (1.1), which is to be expected, as for $3.0 < R_4 \lesssim 3.3$ we are near the rigid rotor limit.

In the transition region, one finds that, for many cases considered, the second term is the dominant term of the three. The two exceptions are Pd^{108} ($R_4 = 2.4$) and Er^{158} ($R_4 = 2.7$) where the third term is dominant. In the vibration region, the third term is either dominant, or is comparable to the second term, the first term being much smaller. Thus the rigid rotor description is not at all valid in these regions and the second and third terms in our expression for E_{rot} may not be considered to be simply corrections to the rigid rotor formula for $2.23 \lesssim R_4 \lesssim 3.0$. A similar analysis of the relative magnitude of the various terms is also given by Das et al.[38] (as discussed in Section 3.5), but there is no one-to-one correspondence between his terms and our terms.

4.4 Conclusions

We have provided a semi-empirical description of the ground-state bands of 88 even-even deformed nuclei. The energies predicted by our approach provide better fits than other models as evidenced by

our WSS values. Although three parameters are employed, this is more than compensated for by the much improved results, especially for the high spin states. The treatment is completely general in that it is independent of the manner in which the variation in δ takes place (e.g. β -stretching, decrease in pairing energy, etc.). Also, since our expression for E_{rot} is in the form of a polynomial, we do not have to worry about the convergence problem inherent in the power series (or perturbation series) expansions as employed by Harris, Satpathy, and others (as previously discussed).

In addition to the greatly improved fits to the experimental data we have introduced a aprameter, B, whose significance is unique to our approach in that it modifies the nuclear softness, σ. Through it, the role of σ, as discussed in the VMI model, is extended. It is hoped that a microscopic investigation will provide further insight into the concept of nuclear softness.

As a further extension of this work, it would also be of interest to see how the parameter B affects the theoretical predictions of the values of nuclear quadrupole moments and the levels of those nuclear excited states which do not belong to the ground-state band, e.g., β - and γ -vibrational bands.

Appendix A

Proof of Feynman's Theorem

Feynman's Theorem states that if $H(\lambda)$ depends on a parameter λ, and $\psi_m(\lambda)$ is an eigenfunction which is normalized to unity then

$$\frac{\partial E_m(\lambda)}{\partial \lambda} = \left\langle \psi_m(\lambda) \left| \frac{\partial H(\lambda)}{\partial \lambda} \right| \psi_m(\lambda) \right\rangle .$$

Proof:

$$\begin{aligned}
\frac{\partial E_m(\lambda)}{\partial \lambda} &= \frac{\partial}{\partial \lambda} \langle \psi_m(\lambda) | H(\lambda) | \psi_m(\lambda) \rangle \\
&= \langle \psi_m(\lambda) | \frac{\partial H(\lambda)}{\partial \lambda} | \psi_m(\lambda) \rangle \\
&\quad + \langle \psi_m(\lambda) | H(\lambda) | \frac{\partial \psi_m(\lambda)}{\partial \lambda} \rangle \\
&\quad + \langle \frac{\partial \psi_m(\lambda)}{\partial \lambda} | H(\lambda) | \psi_m(\lambda) \rangle \\
&= \langle \psi_m(\lambda) | \frac{\partial H(\lambda)}{\partial \lambda} | \psi_m(\lambda) \rangle \\
&\quad + E_m \left[\langle \psi_m(\lambda) | \frac{\partial \psi_m(\lambda)}{\partial \lambda} \rangle + \langle \frac{\partial \psi_m(\lambda)}{\partial \lambda} | \psi_m(\lambda) \rangle \right] \\
&= \langle \psi_m(\lambda) | H(\lambda) | \psi_m(\lambda) \rangle + E_m \frac{\partial}{\partial \lambda} \langle \psi_m(\lambda) | \psi_m(\lambda) \rangle \\
&= \langle \psi_m(\lambda) | H(\lambda) | \psi_m(\lambda) \rangle
\end{aligned}$$

since $\langle \psi_m(\lambda) | \psi_m(\lambda) \rangle = 1$.

APPENDIX B LIST OF OMEGA AND TERMS OF SERIES FOR EACH SPIN

SPIN	OMEGA	TERM 1	TERM 2	TERM 3
PD 108 R(4)= 2.41				
2	266.524967	171.604859	-239.734050	501.931146
4	334.972228	271.063717	-475.929290	1252.355883
6	382.283171	353.040188	-707.410607	2124.385230
CD 110 R(4)= 2.34				
2	419.139339	3416.344408	-12767.233887	10008.629645
4	453.427345	3998.160403	-16163.844566	13707.926083
6	479.041197	4462.626353	-19060.756515	17077.817887
XE 120 R(4)= 2.47				
2	202.402119	43.591841	216.264263	63.151378
4	271.686123	78.543457	523.048501	205.018209
6	323.486315	111.349155	882.891494	412.046532
8	366.113884	142.628888	1279.935809	676.063370
XE 122 R(4)= 2.50				
2	210.822905	64.942421	209.458252	54.252241
4	287.191109	120.513271	529.489155	186.823059
6	344.437020	173.345487	913.426414	386.532315
8	391.602358	224.069883	1342.393961	645.844113
XE 124 R(4)= 2.48				
2	223.844533	61.703169	228.353834	61.775849
4	303.314818	113.292652	568.132603	208.260670
6	362.822384	162.107323	972.417657	426.391995
8	411.826915	208.854550	1422.043924	707.769198
XE 126 R(4)= 2.44				
2	238.003245	29.793112	270.616142	88.107053
4	315.287426	52.283351	629.107718	271.334965
6	372.984075	73.169603	1041.539945	531.422965
8	420.432229	92.969847	1491.739748	857.952333
XE 128 R(4)= 2.34				
2	259.751779	66.916996	-2.113211	379.196357
4	325.914956	105.348403	-4.174267	939.825441
6	373.050004	138.023639	-6.259916	1613.237202
XE 130 R(4)= 2.25				
2	298.411674	227.799150	-800.925064	1107.559223
4	355.503330	323.301581	-1354.177139	2230.891329
6	395.645641	400.436126	-1866.653086	3422.388137
8	427.659371	467.860592	-2357.427987	4671.923166
BA 124 R(4)= 2.83				
2	165.582757	128.272791	90.236432	10.269777
4	249.453092	291.126746	308.534161	52.900055
6	315.167305	464.714945	622.243052	134.792332
BA 126 R(4)= 2.78				
2	182.204226	139.349149	101.231509	11.821365
4	273.727033	314.501564	343.236483	60.214988
6	345.301683	500.477804	649.027350	152.485527
8	405.495285	690.174721	1115.829402	289.986032
CE 128 R(4)= 2.93				
2	154.527274	140.856894	51.539069	14.617936
4	238.604454	335.834196	189.739075	83.095916
6	303.931032	544.901384	392.144630	218.758871
8	358.202779	756.877724	641.960033	422.066903
CE 130 R(4)= 2.83				
2	181.227808	142.501130	83.847824	24.856246
4	269.404170	314.903333	275.442037	121.381764
6	336.422880	491.065789	536.381579	295.174128
8	391.644410	665.506797	846.239082	542.130602
CE 132 R(4)= 2.64				
2	218.979847	120.064199	137.336845	67.691492
4	305.219079	233.253638	371.885646	255.484106

	6	368.704571	340.378495	655.556228	544.040541
	8	420.314518	442.337596	971.175140	918.786790
CE 134 R(4)= 2.56					
	2	265.383475	96.601827	251.055415	60.200492
	4	364.986543	182.722192	653.098253	215.383260
	6	439.815963	265.325851	1142.775519	454.138674
	8	501.531874	345.012371	1694.507833	767.889432
CE 136 R(4)= 2.38					
	2	330.536170	134.752441	-97.148272	514.396862
	4	416.266086	213.717813	-194.039970	1293.917144
	6	476.631844	280.197633	-291.291140	2224.101328
SM 150 R(4)= 2.35					
	2	202.108306	469.753409	-1549.127516	1409.435009
	4	234.926256	634.694496	-2432.928550	2572.969493
	6	257.413397	762.015683	-3200.575830	3708.796395
SM 152 R(4)= 3.01					
	2	92.920498	92.160120	22.488711	7.165875
	4	147.436404	232.022144	89.834711	45.419469
	6	190.541693	387.525259	193.909927	126.701924
	8	226.562474	547.893224	325.982221	253.264919
	10	257.718114	708.940623	479.805050	424.036171
SM 154 R(4)= 3.26					
	2	65.637890	77.498046	3.820827	.039004
	4	116.535506	244.285705	21.382986	.387551
	6	164.542243	487.008286	60.190202	1.540301
	8	210.236994	795.059812	125.551232	4.105184
GD 152 R(4)= 2.20					
	2	186.420782	-61.266163	263.682011	137.733028
	4	234.120525	-96.629784	522.295144	342.626750
	6	269.828156	-128.353159	799.575205	604.522365
GD 154 R(4)= 3.02					
	2	93.824543	92.266915	25.504418	5.273850
	4	149.495327	234.243913	103.168922	33.991664
	6	194.167307	395.153009	226.044716	96.731156
	8	231.926927	563.787482	385.229523	196.909491
	10	264.870667	735.327505	573.809371	334.963445
GD 156 R(4)= 3.24					
	2	71.553946	84.660304	1.555979	2.712060
	4	124.253748	255.288727	8.147635	24.660559
	6	169.934202	477.500946	20.842292	86.275626
	8	209.666302	726.892045	39.146145	199.930697
GD 158 R(4)= 3.29					
	2	64.665719	79.442458	-2.559125	2.513840
	4	114.972758	251.127546	-14.383152	25.120081
	6	159.578631	483.786696	-38.458596	93.226670
	8	198.559529	749.007076	-74.086900	223.462251
GD 160 R(4)= 3.28					
	2	60.775409	72.240045	2.904030	.024455
	4	108.421239	229.906344	16.487719	.247693
	6	153.734293	462.235965	47.003450	1.001241
	8	197.172531	760.352612	99.164607	2.709201
DY 154 R(4)= 2.23					
	2	181.711582	.340677	-22.172763	358.259031
	4	221.148507	.504598	-39.969069	785.964189
	6	249.700247	.643303	-57.534575	1277.446142
	8	272.772197	.767675	-75.001906	1819.144163
	10	292.426921	.882291	-92.411080	2402.902309
DY 156 R(4)= 2.92					
	2	102.239644	88.861529	44.050966	4.976590
	4	158.067385	212.402519	162.788701	28.433060
	6	202.462921	348.470582	342.084931	76.530848
	8	240.068547	489.943109	570.299834	151.284903
DY 158 R(4)= 3.20					
	2	79.024405	91.071866	4.014413	4.053068

	4	134.281495	262.962671	19.696399	33.791371
	6	180.577262	475.540922	47.899120	110.507918
	8	220.044583	706.127504	86.670316	243.659787
	10	254.347720	943.446907	133.851082	434.963219
DY 160 R(4)= 3.28					
	2	70.192387	84.381278	.470730	1.850159
	4	123.990887	263.296822	2.594606	18.013913
	6	172.267317	508.243535	6.958423	67.121331
	8	215.336960	794.151278	13.591203	163.879003
	10	253.833804	1103.481358	22.261301	316.407549
DY 162 R(4)= 3.28					
	2	65.549067	78.036090	2.608198	.432967
	4	116.559432	246.750092	14.665041	4.328913
	6	164.278259	490.142837	41.056439	17.080851
	8	209.029011	793.552393	84.578640	44.772918
DY 164 R(4)= 3.30					
	2	59.712555	72.824485	-.904546	1.479854
	4	106.646867	232.296313	-5.153210	15.057357
	6	149.454169	456.207251	-14.182660	58.074900
	8	187.995260	721.838290	-28.227621	145.393146
ER 156 R(4)= 2.32					
	2	195.838612	-34.135109	258.663654	119.984555
	4	249.756545	-55.518586	536.526375	317.394626
	6	290.000391	-74.851725	839.916661	576.934349
	8	323.089105	-92.907179	1161.468359	888.835337
ER 158 R(4)= 2.74					
	2	137.317670	119.018630	14.297903	57.655582
	4	197.262380	245.612594	42.386336	245.534617
	6	239.722530	362.726932	76.071089	535.514297
	8	273.344223	471.608795	112.777580	905.264014
	10	301.544433	573.937664	151.407370	1340.728733
	12	326.040917	670.974951	191.385658	1832.416018
ER 160 R(4)= 3.09					
	2	99.246775	115.545344	-5.435397	15.124773
	4	162.177727	308.533312	-23.716777	107.842039
	6	209.931371	516.980694	-51.441552	302.783788
	8	248.163635	722.430259	-84.976036	591.256357
	10	280.147399	920.646344	-122.248017	960.217537
	12	307.767330	1111.129498	-162.087525	1398.663761
ER 162 R(4)= 3.24					
	2	81.388405	97.577224	-1.295792	4.611977
	4	140.915742	292.511137	-6.725526	41.445278
	6	191.198664	538.509156	-16.799730	140.467685
	8	233.863491	805.653395	-30.742206	314.403032
	10	270.650343	1079.047466	-47.651161	563.989918
ER 164 R(4)= 3.27					
	2	73.523194	86.287102	4.259415	.848237
	4	129.085405	265.981504	23.051984	8.059874
	6	179.703756	515.479571	62.193980	30.272499
	8	226.054184	815.684575	123.798258	75.800112
	10	268.667212	1152.196329	207.836055	151.244059
ER 166 R(4)= 3.29					
	2	65.450542	79.605148	-1.062662	2.028096
	4	116.176562	250.813956	-5.943070	20.133054
	6	161.628369	485.455960	-16.003244	75.423369
	8	201.908553	757.572309	-31.197472	183.676857
ER 168 R(4)= 3.31					
	2	64.918457	79.020114	0.000000	.732650
	4	116.913153	256.287852	0.000000	7.706860
	6	166.098798	517.290203	0.000000	31.397127
ER 170 R(4)= 3.30					
	2	64.218712	77.738211	.741554	.535744
	4	115.426724	251.144746	4.306031	5.591599
	6	163.993356	506.946522	12.349142	22.783244

YB 158 R(4)= 2.33				
	2	210.483581	157.395361	-399.669271	600.217918
	4	256.473814	233.690760	-723.061399	1323.147258
	6	288.376119	295.443246	-1027.836336	2114.819149
YB 160 R(4)= 2.63				
	2	162.160691	77.878696	129.746213	35.125678
	4	226.555472	152.011559	353.819365	133.826031
	6	274.821685	223.681206	631.554630	289.765291
	8	314.522542	292.975730	946.701921	497.106541
YB 162 R(4)= 2.92				
	2	125.694559	131.283760	-2.500945	36.803573
	4	190.919276	302.884888	-8.764033	195.894925
	6	237.635309	469.245360	-16.900035	470.184110
	8	274.491061	626.086673	-26.045896	837.022040
	10	305.244330	774.235817	-35.817684	1280.013325
YB 164 R(4)= 3.13				
	2	97.041440	113.004929	-3.162089	12.326466
	4	160.160631	307.818319	-14.215747	91.460324
	6	208.988953	524.118610	-31.584381	265.156923
	8	248.460272	740.792939	-53.072953	529.709154
	10	281.641653	951.867921	-77.302411	874.575266
YB 166 R(4)= 3.24				
	2	82.150647	99.970988	-4.323729	5.828090
	4	142.341484	300.133851	-22.491580	52.530103
	6	192.364464	548.155155	-55.514030	175.220655
	8	234.136118	812.061326	-100.099231	384.552776
	10	269.726894	1077.706504	-153.037638	677.297234
	12	300.700437	1339.433868	-212.045672	1046.214770
YB 168 R(4)= 3.26				
	2	70.417161	85.458836	-1.489443	2.851955
	4	124.036983	265.156534	-8.140327	27.455754
	6	171.006763	503.995342	-21.331831	99.193165
	8	211.844531	773.453949	-40.554593	233.613332
	10	247.570400	1056.324576	-64.726843	435.736028
YB 170 R(4)= 3.30				
	2	68.377611	82.279876	1.075513	.987926
	4	121.838272	261.236130	6.084505	9.958724
	6	171.315888	516.488984	16.914805	38.927753
	8	216.877108	827.738601	34.317465	99.982415
	10	258.736874	1178.100757	58.270540	202.535929
YB 172 R(4)= 3.31				
	2	64.173862	79.931909	-3.746581	2.211951
	4	115.607358	259.402879	-21.903676	23.296185
	6	162.240300	510.882844	-60.539218	90.360339
	8	203.518293	803.916131	-119.500835	223.746732
	10	239.875279	1116.798507	-195.666764	431.802214
YB 174 R(4)= 3.29				
	2	62.157393	75.725413	0.000000	.602303
	4	112.149193	246.517854	0.000000	6.383049
	6	159.731699	500.078629	0.000000	26.266850
	8	204.690852	821.207558	0.000000	70.833267
YB 176 R(4)= 3.29				
	2	66.725835	82.211217	-2.095845	1.624187
	4	120.107559	266.369024	-12.223290	17.050665
	6	169.341517	529.504438	-34.258429	67.377323
	8	213.923171	845.003865	-69.063819	171.589725
HF 166 R(4)= 2.97				
	2	122.710016	147.017381	-40.819638	50.828715
	4	187.642123	343.771077	-145.955386	277.914010
	6	232.030686	525.653431	-275.972229	649.786642
	8	266.154367	691.633228	-416.514285	1124.924750
	10	294.201699	845.042309	-562.554550	1679.460496
	12	318.216289	986.687239	-711.876274	2298.738816
HF 168 R(4)= 3.11				

	2	97.837363	115.360999	-8.393331	16.139711
	4	159.940080	308.293347	-36.668407	115.267275
	6	206.675126	514.785086	-79.119722	321.387984
	8	243.875476	716.779861	-129.994360	623.087891
	10	274.883619	910.642437	-186.151973	1005.711676
	12	301.599063	1096.249756	-245.872570	1457.460326
HF 170 R(4)= 3.21					
	2	80.759603	103.768231	-18.042518	13.010352
	4	137.698955	301.673629	-89.434802	109.960046
	6	180.744236	519.762534	-202.259061	326.415038
	8	214.487511	731.947741	-338.003068	647.321999
	10	242.216806	933.436121	-486.774068	1052.760113
	12	265.846082	1124.440773	-643.584250	1527.683495
	14	286.521241	1306.139926	-805.721751	2061.292520
	16	304.969252	1479.749382	-971.589430	2645.675353
HF 172 R(4)= 3.26					
	2	76.563853	95.975619	-8.278710	6.006898
	4	134.536233	296.341104	-44.916830	57.268023
	6	182.709186	546.555329	-112.505165	194.803461
	8	222.535551	810.797069	-203.277595	428.699269
	10	256.129409	1074.069198	-309.934308	752.303231
	12	285.130160	1331.066276	-427.583344	1155.387763
	14	310.673968	1580.240158	-553.102856	1628.450610
HF 174 R(4)= 3.28					
	2	73.629637	87.506506	1.994425	1.762794
	4	129.296284	269.840489	10.799852	16.762354
	6	179.079897	517.641036	28.694601	61.684802
	8	223.562735	806.740492	55.828752	149.826450
	10	263.466540	1120.433182	91.376829	288.996580
HF 176 R(4)= 3.28					
	2	71.606577	86.452459	.152459	1.703151
	4	127.074786	272.248843	.851992	16.890035
	6	177.333312	530.185590	2.315412	64.055188
	8	222.513187	834.754514	4.574300	158.787395
HF 178 R(4)= 3.29					
	2	75.720317	92.078496	-.884584	1.984518
	4	134.824083	291.923004	-4.993491	19.946892
	6	188.393176	569.985365	-13.623772	76.044186
	8	236.394364	897.444518	-26.916168	188.518434
HF 180 R(4)= 3.31					
	2	75.892203	92.081163	.528279	.706820
	4	136.545765	298.080077	3.076851	7.406844
	6	194.055558	602.044588	8.831817	30.215140
	8	248.281533	985.519719	18.497160	80.965115
W 172 R(4)= 3.07					
	2	97.239605	121.468007	-32.715189	33.243138
	4	153.338668	302.050044	-128.284718	205.558769
	6	192.106235	474.087445	-252.257595	506.401650
	8	221.838381	632.191869	-388.445351	900.484028
	10	246.193300	778.624369	-530.943805	1365.948064
	12	266.986694	915.703272	-677.155980	1889.242621
	14	285.239555	1045.189485	-825.751018	2461.321057
W 174 R(4)= 3.17					
	2	89.546973	108.499084	-9.549789	12.503180
	4	149.655197	303.045515	-44.577575	97.540292
	6	195.813440	518.811047	-99.854703	285.882009
	8	232.722052	732.823401	-167.630734	570.383545
	10	263.485822	939.374478	-243.283574	937.229268
	12	289.956584	1137.601621	-324.220115	1374.512628
W 176 R(4)= 3.21					
	2	87.623864	109.385006	-12.637863	11.115404
	4	149.557308	318.660529	-62.839014	94.333558
	6	197.833941	557.989741	-145.448455	288.827949
	8	236.448427	796.500131	-248.322303	589.360968

	10	268.538328	1027.366866	-363.768662	980.529754
	12	296.060566	1248.746036	-487.470270	1448.631458
W 178 R(4)= 3.29					
	2	85.121622	112.109990	-19.515824	10.168723
	4	150.779832	351.763715	-108.466840	100.110553
	6	202.870773	636.800654	-264.195661	328.083796
	8	244.163584	922.414908	-460.585117	688.383428
	10	278.076048	1196.442129	-680.390084	1158.140300
	12	306.876426	1457.107473	-914.445461	1717.753931
	14	331.982704	1705.279013	-1157.746831	2352.711494
W 180 R(4)= 3.29					
	2	83.604638	109.916565	-17.949882	8.955296
	4	149.079060	349.490763	-101.996940	90.536780
	6	201.940547	641.282620	-253.517413	304.826402
	8	244.193401	937.713694	-448.269280	651.769685
	10	279.000725	1224.089223	-668.579724	1110.656570
	12	308.589767	1497.495252	-904.652466	1662.204865
W 182 R(4)= 3.29					
	2	81.361054	99.432668	-1.852670	2.405053
	4	145.044044	316.006311	-10.496585	24.291706
	6	202.532128	616.146245	-28.577887	92.349409
	8	253.719525	966.949368	-56.183658	227.443733
W 184 R(4)= 3.27					
	2	90.005882	108.554187	0.000000	2.520083
	4	159.167118	339.477897	0.000000	24.645897
	6	221.154341	655.383851	0.000000	91.857178
W 186 R(4)= 3.26					
	2	98.959542	118.984840	0.000000	3.323028
	4	174.019414	367.935492	0.000000	31.775621
	6	240.311884	701.660089	0.000000	115.559121
OS 178 R(4)= 3.02					
	2	102.125917	116.211620	-14.139008	29.206244
	4	158.843057	281.134464	-53.200504	170.924679
	6	199.235629	442.294592	-104.981326	423.057942
	8	230.819168	593.637544	-163.240023	762.112900
	10	256.991677	735.894803	-225.303643	1171.137982
	12	279.503470	870.466427	-289.849562	1638.629032
OS 180 R(4)= 3.09					
	2	106.225375	143.009553	-56.033271	43.671575
	4	168.600235	360.267310	-224.045222	277.152292
	6	210.262297	560.313791	-434.555619	670.395954
	8	241.669590	740.205852	-659.822581	1169.967480
	10	267.167555	904.640309	-891.481440	1747.513874
	12	288.823282	1057.238330	-1126.309514	2386.791934
OS 182 R(4)= 3.15					
	2	101.930046	128.601386	-23.721544	21.042109
	4	168.819365	352.765197	-107.771287	158.332265
	6	217.747647	586.878434	-231.257963	438.222087
	8	255.817534	810.031348	-374.996496	834.836855
	10	287.094665	1020.214412	-530.043669	1324.283199
OS 184 R(4)= 3.20					
	2	95.516733	109.454441	7.746873	2.578549
	4	163.882174	322.209172	39.127602	22.345210
	6	223.246256	597.919664	98.910098	76.947465
	8	275.478217	910.435105	185.844568	178.404840
OS 186 R(4)= 3.16					
	2	108.474084	121.205081	12.564065	3.337422
	4	183.362965	346.331578	60.685797	27.249192
	6	247.207977	629.497210	148.709816	90.023742
	8	302.827086	944.622494	273.360870	202.715057
OS 188 R(4)= 3.08					
	2	120.374643	127.174863	21.653877	6.019434
	4	196.971825	340.517528	94.873071	43.155089
	6	259.604492	591.501241	217.203712	130.216011

	8	312.865125	859.103650	380.191233	274.690851
OS 190 R(4)= 2.93					
	2	138.882801	122.989731	58.407718	4.950718
	4	216.437656	298.701611	221.066688	29.201528
	6	278.761324	495.492385	472.304837	80.353419
	8	331.931128	702.534870	797.386326	161.534843
PT 182 R(4)= 2.71					
	2	108.249664	71.490758	72.024171	10.604002
	4	158.253967	152.793894	225.041397	48.437530
	6	196.728882	236.120142	432.317653	115.674081
	8	228.823038	319.445138	680.295878	211.720326
PT 184 R(4)= 2.68					
	2	111.105157	65.551086	81.848860	13.706609
	4	159.793980	135.591444	243.495661	58.645517
	6	196.959394	205.998879	455.974003	135.362997
	8	227.842477	275.664503	705.852681	242.399625
	10	254.629083	344.292427	985.221781	378.116119
PT 186 R(4)= 2.56					
	2	124.045462	63.250454	100.713073	19.708177
	4	175.319593	126.346377	284.337779	78.640225
	6	214.158917	188.527282	518.264771	175.092514
	8	246.311831	249.386206	788.495142	306.382548
	10	274.135975	308.911410	1087.029430	470.096550
PT 188 R(4)= 2.52					
	2	169.499971	49.619127	170.556669	45.086386
	4	230.317622	91.614497	427.899054	153.700607
	6	275.883200	131.449936	735.420258	316.422919
PT 190 R(4)= 2.51					
	2	184.075807	45.905985	191.726071	53.618043
	4	248.391314	83.588917	471.086056	177.774607
	6	296.516354	119.116847	801.377213	361.009568
	8	336.134479	153.074205	1167.428316	596.178564
	10	370.307764	185.781096	1560.915834	878.163485
PT 192 R(4)= 2.48					
	2	198.645738	46.832416	209.063721	59.490214
	4	267.482151	84.913682	510.417248	195.572483
	6	318.972234	120.751856	865.564748	395.494129
	8	361.353877	154.972092	1258.459018	651.417462
PT 194 R(4)= 2.47					
	2	203.497231	35.217261	224.039836	68.977577
	4	271.429030	62.654340	531.641183	218.322801
	6	322.175953	88.272376	889.055960	433.357588
	8	363.920725	112.629498	1281.356866	705.506590
TH 228 R(4)= 3.25					
	2	46.296690	55.192125	-.001985	2.266496
	4	80.242107	165.798990	-.010333	20.453349
	6	109.209737	307.114243	-.026050	70.177934
TH 232 R(4)= 3.27					
	2	40.234760	47.250340	2.293467	.460314
	4	70.674054	145.788111	12.429917	4.382159
	6	98.426780	282.766961	33.575931	16.485468
	8	123.854582	447.740376	66.899744	41.332956
	10	147.242147	632.799882	112.404747	82.561338
U 232 R(4)= 3.29					
	2	38.638768	46.654824	-.001154	1.003011
	4	68.474277	146.522707	-.006421	9.892871
	6	95.369106	284.227073	-.017348	37.225741
U 234 R(4)= 3.30					
	2	35.356732	42.878378	0.000000	.636897
	4	63.192574	136.970240	0.000000	6.498978
	6	88.905036	271.110818	0.000000	25.461639
U 236 R(4)= 3.28					
	2	36.700070	44.318574	.630308	.010377
	4	66.243092	144.386521	4.862693	.110141

	6	94.929400	296.519643	14.369467	.464505
U 238 R(4)= 3.31					
	2	36.424009	44.507155	-.236929	.420990
	4	65.776281	145.141954	-1.395284	4.477117
	6	93.649321	294.214181	-4.026880	18.396693
	8	119.858723	481.940728	-8.442358	49.362807
	10	144.315767	698.685961	-14.736567	103.747250
	12	167.055887	936.219977	-22.858102	186.280916
PU 238 R(4)= 3.31					
	2	35.915143	43.727524	-.000927	.390086
	4	64.713063	141.965762	-.005420	4.111664
	6	91.999693	286.927685	-.015574	16.795609
	8	117.649648	469.224803	-.032569	44.917209
PU 240 R(4)= 3.30					
	2	34.914563	42.544032	.008512	.316524
	4	63.034579	138.670199	.050092	3.362753
	6	89.860247	281.812750	.145122	13.888330
CM 242 R(4)= 3.29					
	2	34.268805	41.395872	0.000000	.862007
	4	60.780242	130.221885	0.000000	8.530303
	6	84.733682	253.087842	0.000000	32.221005
CM 244 R(4)= 3.32					
	2	34.965981	42.608302	-.000855	.325119
	4	63.118123	138.838827	-.005029	3.452038
	6	89.955351	282.004985	-.014558	14.241878
	8	115.358521	463.769456	-.030703	38.517490
CM 248 R(4)= 3.31					
	2	35.346787	43.104140	-.000883	.280978
	4	63.907484	140.903743	-.005220	3.002475
	6	91.281854	287.467001	-.015212	12.497131

Appendix C

Listing of Curve-Fitting Program Employed to Obtain Three-Parameter Fit

```
      PROGRAM ENERGY(INPUT,OUTPUT,TAPE4,TAPE6,TAPE8,TAPE9,TAPE10,TAPE5=I
     1NPUT,TAPE7=OUTPUT)
C     M         =NUMBER OF PARAMETERS
C     L4        =NUMBER OF ITERATIONS ALLOWED
C     N0        =INITIAL CHANNEL OF THE GROUP
C     N         =NUMBER OF CHANNELS IN THE GROUP
C
      DIMENSION Z(256),FM(256),FC(256),DF(256),ERR(256),B(3 ),B1(3 ),
     1B2(3 ,3 ),DC(2183),S(6),DAT(256),S1(6),P(6),PE(6),PH(6),
     2PHE(6),PW(6),PWE(6),A(6),AE(6),ABC(2),W(4),
     3DATA(256)
      DIMENSION VMI(25)
      COMMON/DATA/ABC,W
      COMMON DC
      EQUIVALENCE (Z,DC),(FM,DC(257)),(FC,DC(513)),(DF,DC(769)),
     1(ERR,DC(1025)),(B1,DC(1281)),(B2,DC(1301)),(N,DC(1701)),
     2(L4,DC(1703)),(Q1,DC(1704)),(Q2,DC(1705)),(M,DC(1706)),
     3(B,DC(1719)),(I,DC(2180)),(L,DC(2179)),(B44,DC(2182)),
     4(TERM1,DC(1707)),(TERM2,DC(1708)),(TERM3,DC(1709)),(OMEGA,DC(1710
     5)),(WIEF,DC(1715)),(DELTA,DC(1716))
      EQUIVALENCE(SA,DC(1711)),(SIGMA,DC(1712)),(RAT,DC(1713))
      EQUIVALENCE(ICTOR,DC(1714)),(PAGE,DC(1715)),(SPAGE,DC(1716)),(NOM,
     1DC(1717)),(FNO,DC(1718))
      DATA(ABC=2HNO,3HYES),(W=1H ,1HC,1H*,1HM)
C
C     Q1 AND Q2 ARE THE PRECISIONS OF THE CURVE FITTING
C
      PAGE=0.0
      WRITE(7,449)
  449 FORMAT(1H1)
      WRITE(7,500)
  500 FORMAT(31X,49HTABLE I    COMPARISON OF ROTATIONAL ENERGY LEVELS///
     1)
      WRITE(7,575)
  575 FORMAT(1X,128HFOR EACH NUCLEUS THE FIRST ROW CONTAINS THE ENERGIES
     1 OBTAINED WITH OUR MODEL. THE SECOND ROW GIVES THE EXPERIMENTAL EN
     2ERGIES AND/1X,125HTHE THIRD ROW GIVES ENERGIES PREDICTED BY THE VM
     3I MODEL. ALL ENERGIES ARE IN KEV. (THE VALUES IN THE SECOND AND TH
     4IRD ROW ARE/1X,44HTAKEN FROM REFERENCE 16.) R(4)=E(I=4)/E(I=2)//)
      WRITE(7,501)
  501 FORMAT(3X,4HSPIN,6X,1H2,8X,1H4,8X,1H6,8X,1H8,7X,2H10,7X,2H12,7X,2H
     114,7X,2H16,7X,2H18,7X,2H20,7X,2H22,7X,2H24,7X,2H26/)
      WRITE(8,520)
  520 FORMAT(22X,63HTABLE III   LIST OF PARAMETERS EMPLOYED FOR THREE-PA
     1RAMETER FIT/34X,41HAND THE SOFTNESS SIGMA DERIVED FROM THEM ///)
      WRITE(8,576)
  576 FORMAT(25X,58HIS IN UNITS OF 1/KEV,B IN 1/(KEV**2), AND C IN 1/(KE
     1V**3)./22X,52HSIGMA=(2  C -.9(B**2))/(     **4). R(4)=E(I=4)/E(I=2)
     2//)
      WRITE(8,521)
  521 FORMAT(22X,7HNUCLEUS,7X,4HR(4),13X,4H    ,8X,4HB   ,10X,4HC   ,12X
     1,5HSIGMA//)
      WRITE(9,540)
  540 FORMAT(29X,37HTABLE II     COMPARISON OF WSS VALUES///)
      WRITE(9,541)
```

```
      541 FORMAT(22X,7HNUCLEUS,10X,21HOUR MSS       VMI MSS,11X,4HR(4)//)
          WRITE(4,595)
      595 FORMAT(31X,59HAPPENDIX B  LIST OF OMEGA AND TERMS OF SERIES FOR EA
     1CH SPIN//17X,4HSPIN,7X,5HOMEGA,17X,6HTERM 1,15X,6HTERM 2,15X,6HTER
     2M 3//)
          WRITE(10,571)
      571 FORMAT(1X,48HTABLE IV    LIST OF EFFECTIVE MOMENTS OF INERTIA/1X,3
     10HAND DELTA. (DELTA = (  -  )/ )/)
     1000 CONTINUE
          READ(5,399)CASE,NAME
      399 FORMAT(A2,I3)
          WRITE(6,188)
      188 FORMAT(1H1)
          WRITE(6,340)CASE,NAME
      340 FORMAT(1X,A2,I3)
          READ(5,7)Q1,Q2
        7 FORMAT(2F5.3)
          WRITE(6,8) Q1,Q2
        8 FORMAT (1X,4HQ1 = ,F5.3,5X,4HQ2 = ,F5.3)
          READ (5,939) NO,N    ,L4
          READ(5,910)(DAT(I),I=1,N )
          RAT=DAT(2)/DAT(1)
          READ(5,910)(VMI(I),I=1,N)
          L4=100
          WRITE(4,580)CASE,NAME,RAT
          WRITE(10,573)CASE,NAME,RAT
      573 FORMAT(//1X,A2,I3,4X,7H(R(4) =,F5.2,1H))
      580 FORMAT(1X,A2,1X,I3,1X,5HR(4)=,F5.2)
C     VMI(I) REFER TO VALUES CALCULATED BY OTHERS FOR THE SAME NUCLEUS
      910 FORMAT (12X,10F6.1)
          WRITE(6,3)
        3 FORMAT(2X,13H INITIAL DATA//)
          WRITE(6,931)(DAT(I),I=1,N )
      931 FORMAT(13X,10F9.1)
      939 FORMAT (3I3)
          WRITE(6,10)NO,N
       10 FORMAT(1X20H*INITIAL CHANEL=    I3,25H   *NUMBER OF CHANNELS=  I3)
          M=3
          DO 115 I=1,N
          KNO=I+NO
          Z(I)=(KNO-1)
          KNOI=((I+NO)-1)
          FM(I)=DAT(KNOI)
      115 ERR(I)=SQRT(FM(I))
          WRITE(6,931)(ERR(I),I=1,N)
          READ(5,89)B
       89 FORMAT(2X,3E12.5)
          WRITE(6,89)B
C     B44 = 0 REFERS TO GAMMA VIBRATIONAL STATES
C     B44 = 1 REFERS TO GROUND STATE BANDS
          READ(5,77)B44
       77 FORMAT(10F6.1)
          CALL CURFIT
          WRITE(6,188)
          WRITE (6,135)
```

```
      135 FORMAT(1X,11H PARAMETERS//3X,1HJ,10X,4HB(J),27X,6HERRORS//)
          WRITE (6,140)(J,B(J),B1(J),J=1,3)
      140 FORMAT(3X,I2,5X,E16.6,15X,E16.6 /)
          CALL PLOTB(NO,N,FC,FM)
115       WRITE(6,935)  (FC(KJ),KJ=1,N)
     9461 FORMAT (5H###N=,I5,5H/#NC=,I5,2H//)
          WRITE(6,936)(FM(KJ),KJ=1,N)
          N1=N+1
          N5=N+5
120       WRITE(6,937)(FC(KJ),KJ=N1,N5)
          WRITE(6,938)(VMI(KJ),KJ=1,N)
          WSS=0.0
          DO 835 I=1,N
      835 WSS=WSS+((VMI(I)-DAT(I))**2)/(DAT(I))
125       SIGMA=(2.*B(1)*B(3)-9.*(B(2)**2))/(B(1)**4)
          PAGE=PAGE+1.
          DO 710 NOM=1,9
          FNO=FLOAT(NOM)
          SPAGE=(PAGE+1.)/10.
130       IF(SPAGE-FNO)710,705,710
      705 WRITE(7,449)
      710 CONTINUE
          WRITE(7,502)CASE,NAME,RAT
          WRITE(8,522)CASE,NAME,RAT,B(1),B(2),B(3),SIGMA
135       WRITE(9,542)CASE,NAME,SA,WSS,RAT
      502 FORMAT(//1X,A2,1X,I3,4X,7H(R(4) =,F5.2,1H))
      522 FORMAT(23X,A2,1X,I3,5X,F6.2,7X,E9.3,5X,E9.3,5X,E9.3,9X,E9.3)
      542 FORMAT(23X,A2,1X,I3,9X,E9.3,5X,E9.3,9X,F5.2)
          WRITE(7,503)(FC(I),I=1,N5)
140   503 FORMAT(4X,3HOUR,13F9.1)
          WRITE(7,504)(FM(I),I=1,N)
      504 FORMAT(4X,3HEXP,8F9.1)
          WRITE(7,505)(VMI(I),I=1,N)
      505 FORMAT(4X,3HVMI,8F9.1)
145       WRITE(6,836)WSS
      836 FORMAT(7H WSS = ,F15.6)
      935 FORMAT(15H FC(I),I=1,N,= ,5F15.6)
      936 FORMAT(15H FM(I),I=1,N,= ,5F15.6)
      937 FORMAT(19H FC(I),I=N+1,N+5,= ,5F15.6)
150   938 FORMAT(16H VMI(I),I=1,N,= ,5F15.6)
          READ (5,16) JO
       16 FORMAT (I1)
          IF(JO-2) 1000,1001,1001
     1001 STOP
155       END
```

```
      SUBROUTINE CURFIT
C       F O R T R A N  4
      DIMENSION Z(256),FM(256),FC(256),DF(256),ERR(256),X(256),
     18(3 ),B1(3 ),B2(3 ,3 ),GRAD(3 ),D1(3 ),D2(3 ,3 ),DC(2183),B3(3 ,3
     2),AA(3 ),BB(3 ),W1(3 ),W2(3 ),ABC(2),BB(3 ,3 ),W(4)
      COMMON/DATA/ABC,Y
      COMMON DC
      EQUIVALENCE (Z,DC),(FM,DC(257)),(FC,DC(513)),(DF,DC(769)),
     1(ERR,DC(1025)),(B1,DC(1281)),(B2,DC(1301)),(N,DC(1701)),
     2(L4,DC(1703)),(Q1,DC(1704)),(Q2,DC(1705)),(M,DC(1706)),
     5(TERM1,DC(1707)),(TERM2,DC(1708)),(TERM3,DC(1709)),(OMEGA,DC(1710
     6)),(WIEF,DC(1715)),(DELTA,DC(1716)),
     3(B,DC(1719)),(GRAD,DC(1739)),(D1,DC(1759)),(D2,DC(1799)),
     4(I,DC(2180)),(L,DC(2179))  ,(B44,DC(2182))
      EQUIVALENCE(ICTOR,DC(1714))
      EQUIVALENCE(SA,DC(1711))
      DATA(ABC=2HNO,3HYES),(W=1H ,1HC,1H*,1HM)
      B55=0.0
      L1 = 0
      SA = 0.0
      DO 1000 J=1,M
      B1(J)=0.0
      DO 1000 K=1,M
 1000 B2(J,K)=0.0
      DO 100 I = 1, N
      X(I) = ERR(I)**2
      L=1
      CALL FUNC(2)
      DF(I) =FM(I) - FC(I)
      DO 101 J=1,M
      B1(J)=B1(J)-(2.0*DF(I)*D1(J))/X(I)
      DO 101 K=1,M
  101 B2(J,K)=B2(J,K)-(2.0*(DF(I)*D2(J,K)-D1(J)*D1(K)))/X(I)
  100 SA = SA + DF(I)**2/X(I)
      GMOD=0.0
      DO 102 J=1,M
  102 GMOD=GMOD+B1(J)**2
      WRITE(6,243)SA,GMOD
  243 FORMAT(1X,26H*INITIAL VALUE SUM OF SQ.=E13.5,20X,17H*SQ MOD OF GR
     1AD=E13.5)
      WRITE(6,1751)
 1751 FORMAT(14H0 DERIVATIVES-)
      WRITE(6,240)(B1(J),J=1,M)
  240 FORMAT (15X,5(E13.5,8X)/)
      IF (SA - Q1) 110, 110, 200
  110 LE = 1
      GO TO 600
  200 S = 0.0
      GMOD = 0.0
      BMOD = 0.0
      PROD = 0.0
      A2=ABC(1)
      DO 210 J = 1, M
      B1(J) = 0.0
      DO 210 K = 1, M
```

```
      210 B2(J,K) = 0.0
          DO 220 I = 1, N
          L=1
          CALL FUNC(2)
60        DF(I) = FM(I) - FC(I)
          WRITE(6,11140)DF(I)
    11140 FORMAT(7H DF(I)=,E17.7)
          DO 220 J = 1, M
          B1(J) = B1(J) - (2.0*DF(I)*D1(J))/X(I)
65        DO 220 K = 1, M
      220 B2(J,K) = B2(J,K) - (2.0*(DF(I)*D2(J,K) -D1(J)*D1(K)))/X(I)
          WRITE(6,11111)B1
           DO 230 J = 1, M
      230 GRAD(J) = B1(J)
70        L1 = L1 + 1
          CALL EXAM (B2,B1,M,LF)
          WRITE(6,11111)B
          WRITE(6,11113)LF
    11113 FORMAT(4H LF=,I4)
75        IF (LF)  250, 250, 305
     .250 DO 231 II=1,M
          DO 231 JJ=1,M
      231 B3(II,JJ)=B2(II,JJ)
          DO 11130 I=1,3
80  11130 WRITE(6,11131)(B3(I,J),J=1,3)
    11131 FORMAT(9H B3(I,J)=,3E17.7)
          CALL JACOBI (M,B3,1,NR,B2)
          WRITE(6,11111)B
          DO 11132 I=1,M
85  11132 WRITE(6,11131)(B3(I,J),J=1,3)
          DO 235 I = 1,M
      235 B1(I)=B3(I,I)
          A2=ABC(2)
          DO 260 J = 1, M
90    260 D1(J) = 0.0
          DO 270 J = 1, M
          DO 270 K = 1, M
      270 D1(K) = D1(K) + B2(J,K) *GRAD(J)
          DO 275 J = 1, M
95        WRITE(6,11114)B1(J)
    11114 FORMAT(7H B1(J)=,E17.7)
          IF (B1(J))  280, 290, 285
      280 B1(J) = - B1(J)
      285 D1(J) = D1(J)/B1(J)
100       WRITE(6,11111)B
          GO TO 275
      290 D1(J)=0.0
      275 CONTINUE
          DO 295 J = 1, M
105   295 B1(J) = 0.0
          DO 300 J = 1, M
          DO 300 K = 1, M
      300 B1(J) = B1(J) + B2(J,K)*D1(K)
      305 DO 310 J=1,M
110       GMOD = GMOD + GRAD(J)**2
```

```
                BMOD = BMOD + B1(J)**2
          310   PROD = PROD + GRAD(J)*B1(J)
                WRITE(6,11112)BMOD,GMOD
        11112   FORMAT(9H ACHECK,  ,2E17.7)
115             WRITE(6,11111)B
                WRITE(6,11115)GMOD
        11115   FORMAT(6H GMOD=,E17.7)
                IF (GMOD - Q2)  315, 315, 320
          315   LE = 2
120             WRITE(6,11111)BMOD,GMOD
                GO TO 600
          320   C=PROD/SQRT(BMOD*GMOD)
                WRITE(6,11116)C
        11116   FORMAT(3H C=,E17.7)
125             IF (C) 335, 335, 400
          335   LE = 4
                GO TO 600
          400   LD = 0
                L3 = 0
130             DO 410 J = 1, M
          410   GRAD(J) = B(J) - B1(J)
                WRITE(6,11111)B
        11111   FORMAT(6H CHECK,20X,4E17.8)
          450   DO 420 I = 1, N
135             L=2
                CALL FUNC (1)
                DF(I) = FM(I) - FC(I)
          420   S = S + DF(I)**2/X(I)
                WRITE(6,11118)SA,S
140     11118   FORMAT(4H SA=,E17.7,3H S=,E17.7)
                IF (SA - S) 435, 500, 500
          435   LD = LD + 1
          430   DO440 J = 1, M
                B1(J) = B1(J)/2.0
145       440   GRAD(J) = B(J) - B1(J)
                WRITE(6,11111)B
                WRITE(6,11111) GRAD
                WRITE(6,11111)B1
                S = 0.0
150             L3 = L3 + 1
                WRITE(6,11120)L3
        11120   FORMAT(4H L3=,I4)
                IF (L3 -  8) 450,460,460
          460   LE = 5
155             WRITE(6,11119)LD
        11119   FORMAT(4H LD=,I4)
                GO TO 600
          500   IF (LD) 505, 505, 506
          506   LD = 0
160             GO TO 430
          505   DO 510 J = 1, M
          510   B(J) = GRAD(J)
                B55=B55+1.
        C       FOLLOWING CARDS ARE FOR ROTATIONAL LEVELS
165             WRITE(6,11111)B55
```

```
      IF(855)22,22,23
   23 I6=0
      DO 20 I=1,N
      IF(844)18,18,19
   18 CONTINUE
      FI=(FLOAT(I))+2.
      FIA=SQRT((FI)*(FI+1.))-SQRT(6.)
      GO TO 30
   19 FI=FLOAT(I)
      FII=(2.*FI)*((2.*FI)+1.)
      FIA=SQRT(FII)
   30 ETA=FIA
      R55=(B(2)**3)/(4.*(B(3)**3))-B(1)*B(2)/(4.*(B(3)**2))-ETA/(2.*B(3)
     1)
      Q55=B(1)/(2.*B(3))-(3.*(B(2)**2))/(4.*(B(3)**2))
      RQ=(R55**2)/(4.)+(Q55**3)/(27.)
      IF(RQ)10,20,20
   10 DO 11 J=1,M
      GRAD(J)=B(J)+B1(J)
      B1(J)=B1(J)/2.
   11 B(J)=GRAD(J)-B1(J)
      I6=1
      GO TO 30
   20 CONTINUE
      IF(I6-1)22,23,23
   22 CONTINUE
C     UPPER CARDS ARE FOR ROTATIONAL LEVELS
      SA = S
      WRITE(6,11121)SA
11121 FORMAT(4H SA=,E17.7)
      IF (SA - Q1) 507, 507, 530
  507 LE = 1
      WRITE(6,11122)L4
11122 FORMAT(4H L4=,I4)
      GO TO 600
  530 IF (L4) 200, 200, 900
  900 WRITE(6,920)L1,A2,L3,S,GMOD,(B(J),J=1,M)
  920 FORMAT(//,15H ITERATION NO.=I5,10X,43H   TRANSFORMATION MADE TO PR
     1INCIPAL AXES = A4,10X,   18H BINARY CHOP USED=I3,6H TIMES/1X,27H W
     2EIGHTED SUM OF SQUARES = E14.7,25X,32H   SQUARE MODULUS OF GRADIEN
     3T = E14.7/20H   PARAMETERS B(J) -/(6E17.8)/)
      IF (L1-L4) 200, 910, 910
  910 LE = 6
      GO TO 600
  600 DO 710 J=1,M
      B1(J) = 0.0
      DO 710 K=1,M
  710 B2(J,K) = 0.0
      L=1
      DO 720 I = 1, N
      CALL FUNC(2)
      WRITE(10,575)WIEF,DELTA
  575 FORMAT(20X,F15.6,5X,F15.6)
      ICTOR=I+I
      WRITE(4,940)ICTOR,OMEGA,TERM1,TERM2,TERM3
```

```
      SUBROUTINE  CURFIT                                                
```

```
      SUBROUTINE FUNC (LX)
C     SUBROUTINE FUNC
      DIMENSION DC(2183),B(3    ,D1(3 ),D2(3 ,3),FC(256),Z(256),E(6)
      DIMENSION R(4,4),Q(4,4),RAD(4,4),PAD(4,4),FAD(4,4),W(4,4),EE(3),
     1EEW(3),WF(4,4),EWE(3),RRAD(4,4)
      COMMON DC
      EQUIVALENCE (Z,DC),(FC,DC(513)),(B,DC(1719)),(D1,DC(1759)),
     1(D2,DC(1799)),(M,DC(1706)),(L,DC(2179)),(I,DC(2180)),
     2 (B44,DC(2182)),
     3(TERM1,DC(1707)),(TERM2,DC(1708)),(TERM3,DC(1709)),(OMEGA,DC(1710
     4)),(WIEF,DC(1715)),(DELTA,DC(1716))
      DO 70 I1 = 1,M
      D1(I1) = 0.0
      DO 70 I2 = 1,M
   70 D2(I1,I2) = 0.0
      IF(B44)78,78,79
   78 CONTINUE
      FI=(FLOAT(I))+2.
      FIA=SQRT((FI)*(FI+1.))-SQRT(6.)
      GO TO 80
   79 FI=FLOAT(I)
      FII=(2.*FI)*((2.*FI)+1.)
      FIA=SQRT(FII)
   80 CONTINUE
      ETA=FIA
      R(4,4)=(B(2)**3)/(4.*(B(3)**3))-B(1)*B(2)/(4.*(B(3)**2))-ETA/(2.*B
     1(3))
      R(4,1)=-B(2)/(4.*(B(3)**2))
      R(4,2)=(3.*(B(2)**2))/(4.*(B(3)**3))-E(1)/(4.*(B(3)**2))
      R(4,3)=(-3.*(B(2)**3))/(4.*(B(3)**4))+B(1)*B(2)/(2.*(B(3)**3))+ETA
     1/(2.*(B(3)**2))
      R(1,1)=0.0
      R(1,2)=-1./(4.*(B(3)**2))
      R(1,3)=B(2)/(2.*(B(3)**3))
      R(2,1)=R(1,2)
      R(2,2)=3.*B(2)/(2.*(B(3)**3))
      R(2,3)=(-9.*(B(2)**2))/(4.*(B(3)**4))+B(1)/(2.*(B(3)**3))
      R(3,1)=R(1,3)
      R(3,2)=R(2,3)
      R(3,3)=(3.*(B(2)**3))/(B(3)**5)-3.*B(1)*B(2) /(2.*(B(3)**4))-ETA/
     1(9(3)**3)
      Q(4,4)=B(1)/(2.*B(3))-(3.*(B(2)**2))/(4.*(B(3)**2))
      Q(4,1) =1./(2.*B(3))
      Q(4,2)=-3.*B(2)/(2.*(B(3)**2))
      Q(4,3)=-B(1)/(2.*(B(3)**2))+(3.*(B(2)**2))/(2.*(B(3)**3))
      Q(1,1)=0.0
      Q(1,2)=0.0
      Q(1,3)=-1./(2.*(B(3)**2))
      Q(2,1)=Q(1,2)
      Q(2,2)=-3./(2.*(B(3)**2))
      Q(2,3)=3.*B(2)/(B(3)**3)
      Q(3,1)=Q(1,3)
      Q(3,2)=Q(2,3)
      Q(3,3)=B(1)/(B(3)**3)-(9.*(B(2)**2))/(2.*(B(3)**4))
```

```
      RAD(4,4)=(R(4,4)**2)/4.+(Q(4,4)**3)/27.
      DO 130 J=1,M
  130 RAD(4,J)=R(4,4)*R(4,J)/2.+(Q(4,4)**2)*Q(4,J)/9.
      DO 131 K=1,M
      DO 131 J=1,M
  131 RAD(K,J)= R(4,K)*R(4,J)/2.+R(K,J)*R(4,4)/2.+(2.*Q(4,4)*Q(4,K
     1)*Q(4,J)+Q(4,4)*Q(4,4)*Q(K,J))/9.
      RRAD(4,4)=SQRT(RAD(4,4))
      DO 132 J=1,M
  132 RRAD(4,J)=(1./(2.*RRAD(4,4)       ))*RAD(4,J)
      DO 133 K=1,M
      DO 133 J=1,M
  133 RRAD(K,J)=(1./(2.*     RRAD(4,4))) *RAD(K,J)-RAD(4,J)*RRAD(4,K)/
     1(2.*(RRAD(4,4)**2))
      PAD(4,4)=-R(4,4)/2.+RRAD(4,4)
      DO 1134 J=1,M
 1134 PAD(4,J)=-R(4,J)/2.+RRAD(4,J)
      DO 134 K=1,M
      DO 134 J=1,M
  134 PAD(K,J)=-R(K,J)/2.+RRAD(K,J)
      FAD(4,4)=R(4,4)/2.+RRAD(4,4)
      DO 135 K=1,M
      DO 135 J=1,M
  135 FAD(K,J)=R(K,J)/2.+RRAD(K,J)
      DO 1135 J=1,M
 1135 FAD(4,J)=R(4,J)/2.+RRAD(4,J)
      A=1./3.
      A2=2./3.
      A5=5.*A
      IF (PAD(4,4))1,2,2
    1 PS = -1.0
      GO TO 4
    2 PS = +1.0
    4 CONTINUE
      IF (FAD(4,4)) 5,6,6
    5 FS=-1.0
      GO TO 7
    6 FS=+1.0
    7 CONTINUE
      W(4,4)=-B(2)/(2.*B(3))+PS*((PS*PAD(4,4))**A)-FS*((FS*FAD(4,4))**A)
      FC(I)=B(1)*(W(4,4)**2)/2.+2.*B(2)*(W(4,4)**3)+3.*B(3)*(W(4,4)**4)
     1/2.
      OMEGA=W(4,4)
      TERM1=B(1)*(W(4,4)**2)/2.
      TERM2=2.*B(2)*(W(4,4)**3)
      TERM3=(3.*B(3)*(W(4,4)**4))/2.
      WIEF=B(1)+3.*B(2)*W(4,4)+2.*B(3)*(W(4,4)**2)
      DELTA=(WIEF-B(1))/WIEF
      GO TO (110,120),LX
  120 CONTINUE
      EE(1)=(W(4,4)**2)/2.
      EE(2)=2.*(W(4,4)**3)
      EE(3)=3.*(W(4,4)**4)/2.
      EW=B(1)*W(4,4)+6.*B(2)*(W(4,4)**2)+6.*B(3)*(W(4,4)**3)
      EWW=B(1)+12.*B(2)*W(4,4)+18.*B(3)*(W(4,4)**2)
```

```
      EEW(1)=W(4,4)
      EEW(2)=6.*(W(4,4)**2)
      EEW(3)=6.*(W(4,4)**3)
      DO 136 K=1,M
115  136 EWE(K)=EEW(K)
      WF(4,4)=-B(2)/(2.*B(3))
      WF(4,1)=0.0
      WF(4,2)=-1./(2.*B(3))
      WF(4,3)=B(2)/(2.*(B(3)**2))
120   WF(1,1)=0.0
      WF(1,2)=0.0
      WF(1,3)=0.0
      WF(2,1)=0.0
      WF(2,2)=0.0
125   WF(2,3)=1./(2.*(B(3)**2))
      WF(3,1)=0.0
      WF(3,2)=WF(2,3)
      WF(3,3)=-B(2)/(B(3)**3)
      DO 137 J=1,M
130  137 W(4,J)=WF(4,J)+PAD(4,J)/(3.*((PS*PAD(4,4))**A2))-FAD(4,J)/(3.*((FS
     1*FAD(4,4))**A2))
      DO 138 K=1,M
      DO 138 J=1,M
     138 W(K,J)=WF(K,J)-2.*PAD(4,J)*PAD(4,K)*PS/(9.*((PS*PAD(4,4))**A5))
135  1+PAD(K,J)/(3.*((PS*PAD(4,4))**A2))+2.*FAD(4,K)*FAD(4,J)*FS/(9.*
     2((FS*FAD(4,4))**A5))-FAD(K,J)/(3.*((FS*FAD(4,4))**A2))
      DO 139 J=1,M
     139 D1(J)=EE(J)+EW*W(4,J)
      DO 140 K=1,M
140   DO 140 J=1,M
     140 D2(K,J)=EEW(K)*W(4,J)+EWE(J)*W(4,K)+EWW*W(4,J)*W(4,K)+EW*W(J,K)
     110 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE EXAM(A,B,M,LF)
C     SUBROUTINE EXAM
C       F O R T R A N  4
      DIMENSION A(3 ,3 ),B(3 ),C(3 )
      DO 80 J=1,M
   80 C(J)=A(J,J)
      IF(A(1,1)) 60,200,70
   60 A(1,1) =-SQRT(-A(1,1))
      GO TO 300
   70 A(1,1) =SQRT(A(1,1))
      GO TO 100
  100 IF(M-1)400,400,110
  110 DO 115 K=2,M
  115 A(1,K)=A(1,K)/(A(1,1)          )
      DO 120 J=2,M
      J1=J-1
      S=A(J,J)
      DO 125 L=1,J1
  125 S=S-A(L,J)**2
      IF (S) 50,200,40
   50 A(J,J) =-SQRT(-S)
      GO TO 300
   40 A(J,J) =SQRT(S)
      GO TO 130
  130 IF(J-M)135,400,400
  135 J2=J+1
      DO 120 K=J2,M
      S=A(J,K)
      DO 145 L=1,J1
  145 S=S-A(L,J)*A(L,K)
  120 A(J,K)=S/A(J,J)
  400 B(1)=B(1)/A(1,1)
      IF(M-1)420,420,405
  405 DO 410 J=2,M
      S=B(J)
      J1=J-1
      DO 415 L=1,J1
  415 S=S-A(L,J)*B(L)
  410 B(J)=S/A(J,J)
  420 B(M)=B(M)/A(M,M)
      J=M-1
  435 IF(J)450,450,425
  425 S=B(J)
      J2=J+1
      DO 430 L=J2,M
  430 S=S-A(J,L)*B(L)
      B(J)=S/A(J,J)
      J=J-1
      GO TO 435
  450 LF=1
      GO TO 460
  200 LF=0
      GO TO 460
  300 LF=-1
```

```
      SUBROUTINE  EXAM                          CDC 6600 FTN V3.0-P296 OPT=1  72/09/18. 18.42.07.      PAGE      2

                  460 DO 465 J=1,M
                      A(J,J)=C(J)
                      IF(J-M)470,475,475
                  470 J2=J+1
60                    DO 465 K=J2,M
                  465 A(J,K)=A(K,J)
                  475 RETURN
                      END
```

```
      SUBROUTINE MATINV(A,N,B,M,DETERM)
C     SUBROUTINE MATINV
C       F O R T R A N  4
C     MATRIX INVERSION WITH ACCOMPANYING SOLUTION OF LINEAR EQUATIONS
      DIMENSION IPIVOT(3 ),A(3, 3 ),B(3 ,1),INDEX(3 ,3),PIVOT(3 )
      EQUIVALENCE (IROW,JROW),(ICOLUM,JCOLUM),(AMAX,T,SWAP)
      DETERM=1.0
      DO 20 J=1,N
   20 IPIVOT(J)=0
      DO 550 I=1,N
      AMAX=0.0
      DO 105 J=1,N
      IF(IPIVOT(J)-1)60,105,60
   60 DO 100 K=1,N
      IF(IPIVOT(K)-1)80,100,740
   80 IF(ABS(AMAX)-ABS(A(J,K)))85,100,100
   85 IROW=J
      ICOLUM=K
      AMAX=A(J,K)
  100 CONTINUE
  105 CONTINUE
      IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
      IF (IROW-ICOLUM)140,260,140
  140 DETERM=-DETERM
      DO 200 L=1,N
      SWAP=A(IROW,L)
      A(IROW,L)=A(ICOLUM,L)
  200 A(ICOLUM,L)=SWAP
      IF(M)260,260,210
  210 DO 250 L=1,M
      SWAP=B(IROW,L)
      B(IROW,L)=B(ICOLUM,L)
  250 B(ICOLUM,L)=SWAP
  260 INDEX(I,1)=IROW
      INDEX(I,2)=ICOLUM
      PIVOT(I)=A(ICOLUM,ICOLUM)
      DETERM=DETERM*PIVOT(I)
      A(ICOLUM,ICOLUM)=1.0
      DO 350 L=1,N
  350 A(ICOLUM,L)=A(ICOLUM,L)/PIVOT(I)
      IF(M)380,380,360
  360 DO 370 L=1,M
  370 B(ICOLUM,L)=B(ICOLUM,L)/PIVOT(I)
  380 DO 550 L1=1,N
      IF (L1-ICOLUM)400,550,400
  400 T=A(L1,ICOLUM)
      A(L1,ICOLUM)=0.0
      DO 450 L=1,N
  450 A(L1,L)=A(L1,L)-A(ICOLUM,L)*T
      IF(M)550,550,460
  460 DO 500 L=1,M
  500 B(L1,L)=B(L1,L)-B(ICOLUM,L)*T
  550 CONTINUE
      DO 710 I=1,N
```

```
      L=N+1-I
      IF(INDEX(L,1)-INDEX(L,2))630,710,630
  630 JROW=INDEX(L,1)
      JCOLUM=INDEX(L,2)
      DO 705 K=1,N
      SWAP=A(K,JROW)
      A(K,JROW)=A(K,JCOLUM)
      A(K,JCOLUM)=SWAP
  705 CONTINUE
  710 CONTINUE
  740 RETURN
      END
```

```
      SUBROUTINE PLOTB (N0,N,AA,BB)
C     SUBROUTINE PLOT B
C     A= LARGEST OF FC AND FM, B= SMALLEST
      DIMENSION X(116),AA(512),BB(512),W(4),ABC(2)
      COMMON/DATA/ABC,W
      DATA(ABC=2HNO,3HYES),(W=1H ,1HC,1H*,1HM)
      A=AA(1)
      B=A
      DO 900 I=1,N
      IF(AA(I)-A)905,905,910
  910 A=AA(I)
  905 IF(BB(I)-A)915,915,920
  920 A=BB(I)
  915 IF(AA(I)-B)930,925,925
  930 B=AA(I)
  925 IF(BB(I)-B)935,900,900
  935 B=BB(I)
  900 CONTINUE
      FACTOR = 1.0
  520 IF (A-B-1000.0) 500, 510, 510
  500 A = 2.0*A
      B = 2.0*B
      FACTOR = 2.0*FACTOR
      GO TO 520
  510 KD = (A-B)/112.0 + 1.0
      KS = IFIX(B) - 2*KD
      WRITE (6,1)
    1  FORMAT(119H0...0.........1.........2.........3.........4.........
     15.........6.........7.........8.........9.........10.,.....,11...)
      DO 100 I=1,N
      DO 110 K=1,116
  110 X(K)=W(1)
      K = AA(I)*FACTOR
      K = (K-KS)/KD
      X(K) = W(2)
      L = BB(I)*FACTOR
      L = (L-KS)/KD
      IF(L-K)120,130,120
  130 X(L)=W(3)
      GO TO 105
  120 X(L)=W(4)
  105 IN0=I+N0-1
  100 WRITE (6,90)IN0,X
   90 FORMAT(1XI3,116A1)
      WRITE (6,1)
      RETURN
      END
```

```
      SUBROUTINE JACOBI (N,Q,JVEC,M,V)
C     SUBPROGRAM FOR DIAGONALIZATION OF MATRIX Q BY SUCCESSIVE ROTATIONS
      DIMENSION Q(3 ,3 ),V(3 ,3 ),X(3 ),IH(3 )
C
C     NEXT 8 STATEMENTS FOR SETTING INITIAL VALUES OF MATRIX V
C
      IF(JVEC) 10,15,10
   10 DO 14 I=1,N
      DO 14 J=1,N
      IF(I-J) 12,11,12
   11 V(I,J)=1.0
      GO TO 14
   12 V(I,J)=0.
   14 CONTINUE
C
   15 M=0
C     NEXT 8 STATEMENTS SCAN FOR LARGEST OFF DIAG. ELEM. IN EACH ROW
C     X(I) CONTAINS LARGEST ELEMENT IN ITH ROW
C     IH(I) HOLDS SECOND SUBSCRIPT DEFINING POSITION OF ELEMENT
C
      MI=N-1
      DO 30 I=1,MI
      X(I)=0.
      MJ=I+1
      DO 30 J=MJ,N
      IF (X(I)-ABS (Q(I,J))) 20,20,30
   20 X(I)=ABS (Q(I,J))
      IH(I)=J
   30 CONTINUE
C
C     NEXT 7 STATEMENTS FIND FOR MAXIMUM OF X(I)S FOR PIVOT ELEMENT
   40 DO 70 I=1,MI
      IF(I-1) 60,60,45
   45 IF (XMAX-X(I)) 60,70,70
   60 XMAX=X(I)
      IP=I
      JP=IH(I)
   70 CONTINUE
C
C     NEXT 2 STATEMENTS TEST FOR XMAX,IF LESS THAN 10**-8,GO TO 1000
C
      EPSI=1.E-8
      IF (XMAX-EPSI) 1000,1000,148
C
  148 M=M+1
C
C     NEXT 11 STATEMENTS FOR COMPUTING TANG,SINE,COSN,Q(I,I),Q(J,J)
C
      IF (Q(IP,IP)-Q(JP,JP)) 150,151,151
  150 TANG =-2.*Q(IP,JP)/(ABS(Q(IP,IP)-Q(JP,JP))+SQRT((Q(IP,IP)-Q(JP,JP).
     1)**2+4.*Q(IP,JP)**2))
      GO TO 160
  151 TANG =+2.*Q(IP,JP)/(ABS(Q(IP,IP)-Q(JP,JP))+SQRT((Q(IP,IP)-Q(JP,JP)
     1)**2+4.*Q(IP,JP)**2))
  160 COSN=1.0/SQRT(1.0+TANG**2)
```

```
                SINE=TANG*COSN
                QII= Q(IP,IP)
                Q(IP,IP)= COSN**2*(QII+TANG*(2.*Q(IP,JP)+TANG*Q(JP,JP)))
                Q(JP,JP)= COSN**2*(Q(JP,JP)-TANG*(2.*Q(IP,JP)-TANG*QII))
60          C
                Q(IP,JP)=0.
            C
            C   NEXT 4 STATEMENTS FOR PSEUDO RANK OF THE EIGENVALUES
                IF (Q(IP,IP)-Q(JP,JP)) 152,153,153
65        152 TEMP=Q(IP,IP)
                Q(IP,IP)=Q(JP,JP)
                Q(JP,JP)=TEMP
            C
            C   NEXT 6 STATEMENTS ADJUST SIN,COS FOR COMPUTATION OF Q(I,K),V(I,K)
70          C
                IF(SINE) 154,155,155
            154 TEMP=+COSN
                GO TO 170
            155 TEMP=-COSN
75          170 COSN=ABS(SINE)
                SINE=TEMP
            C
            C   NEXT 10 STATEMENTS FOR INSPECTING  THE I#S BETWEEN I+1 AND N-1 TO
            C   DETERMINE WHETHER A NEW MAXIMUM VALUE SHOULD BE COMPUTED SINCE
80          C   THE PRESENT MAXIMUM IS IN THE I OR J ROW
            C
            153 DO 350 I=1,MI
                IF (I-IP) 210,350,200
            200 IF (I-JP) 210,350,210
85          210 IF (IH(I)-IP) 230,240,230
            230 IF (IH(I)-JP) 350,240,350
            240 K= IH(I)
                TEMP=Q(I,K)
                Q(I,K)=0.
90              MJ=I+1
                X(I)=0.
            C
            C   NEXT 5 STATEMENTS SEARCH IN DEPLETED ROW FOR NEW MAXIMUM
            C
95              DO 320  J=MJ,N
                IF (X(I)-ABS(Q(I,J))) 300,300,320
            300 X(I)=ABS(Q(I,J))
                IH(I)=J
            320 CONTINUE
100             Q(I,K)=TEMP
            350 CONTINUE
            C
                X(IP)=0.
                X(JP)=0.
105         C
            C   NEXT 30 STATEMENTS FOR CHANGING THE OTHER ELEMENTS OF Q
            C
                DO 530 I=1,N
            C
110             IF (I-IP) 370,530,420
```

```
      370 TEMP=Q(I,IP)
          Q(I,IP)=COSN*TEMP+SINE*Q(I,JP)
          IF (X(I)-ABS(Q(I,IP))) 380,390,390
      380 X(I)=ABS(Q(I,IP))
          IH(I)=IP
      390 Q(I,JP)=-SINE*TEMP+COSN*Q(I,JP)
          IF (X(I)-ABS(Q(I,JP))) 400,530,530
      400 X(I)=ABS(Q(I,JP))
          IH(I)=JP
          GO TO 530
C
      420 IF (I-JP) 430,530,480
      430 TEMP =Q(IP,I)
          Q(IP,I)=COSN*TEMP+SINE*Q(I,JP)
          IF (X(IP)-ABS(Q(IP,I))) 440,450,450
      440 X(IP)=ABS(Q(IP,I))
          IH(IP)=I
      450 Q(I,JP)=-SINE*TEMP+COSN*Q(I,JP)
          IF (X(I)-ABS(Q(I,JP))) 400,530,530
C
      480 TEMP=Q(IP,I)
          Q(IP,I)=COSN*TEMP+SINE*Q(JP,I)
          IF(X(IP)-ABS(Q(IP,I))) 490,500,500
      490 X(IP)=ABS(Q(IP,I))
          IH(IP)=I
      500 Q(JP,I)=-SINE*TEMP+COSN*Q(JP,I)
          IF (X(JP)-ABS(Q(JP,I))) 510,530,530
      510 X(JP)=ABS(Q(JP,I))
          IH(JP)=I
      530 CONTINUE
C
C     NEXT 6 STATEMENTS TEST FOR COMPUTATION OF EIGENVECTORS
C
          IF (JVEC) 540,40,540
      540 DO 550 I=1,N
          TEMP=V(I,IP)
          V(I,IP)= COSN*TEMP+SINE*V(I,JP)
      550 V(I,JP)=-SINE*TEMP+COSN*V(I,JP)
          GO TO 40
     1000 RETURN
          END
```

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